# A Watershed Model for Estimating Suspended Sediment Hydrographs

BY
Hafez Q. Shaheen

Assistant Professor; An-Najah N. University, Nablus

A mathematical model was developed to estimate soil detachment and transport and, consequently, suspended sediment hydrograph for single storms on a watershed basis. In this linear cascade model the sediment graphs are generated by convolution of the exponential detachment equation along with a simplified form of the sediment continuity equation. The verification and calibration of the model was done using measurements from watersheds in the Federal Republic of Germany. The relation between suspended sediment and water discharge was investigated using this model. The result was a well defined loop that can be determined using the rainfall erosivity factor and the critical discharge needed to initiate sediment movement.

## Introduction

Erosion and sedimentation by water embody the processes of detachment, transportation and deposition of soil particles by the erosive agents of raindrop impact and runoff over the soil surface (ASCE, 1975). Detachment is the dislodging of soil particles from the soil mass by the erosive agents. Transportation is the entrainment and movement of sediment from its original location. Sediment travels from upland sources through the streams and may eventually reach the sea. Not all sediment reaches the sea, some of it is deposited on the toe of slopes in reservoirs, and on flood plains along the way. This process is Sedimentation. The three processes, erosion, transportation and sedimentation doesn't occur over the soil surface only, but also in the stream itself.

Erosion and sedimentation can be major problems (ASCE, 1975). Erosion reduces the productivity of cropland. Sediment degrades water quality and may carry soil absorbed polluting chemicals. Depositions in irrigation canals, stream channels, reservoirs, estuaries, harbors and other water conveyance structures reduces the capacity of these structures and require costly removal.

In this paper a model is introduced that estimate the suspended sediment hydrograph resulted from a single storm on a watershed basis. The principles of linear cascade model and time-area diagram were used. A convolution integral of the sediment continuity equation and the exponential detachment equation which include the physical properties of the storm along with the properties of the watershed and its vegetation cover form the idea of the model.

**Physical Description of the erosion and sedimentation process**

The process of erosion and sedimentation in a watershed begins with the first rainfall drop (Fig. 1).

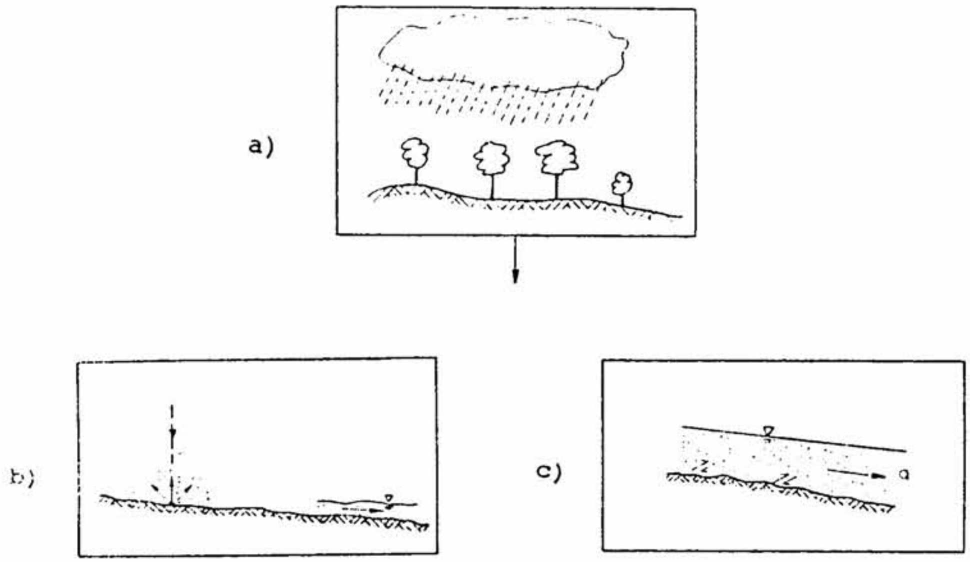


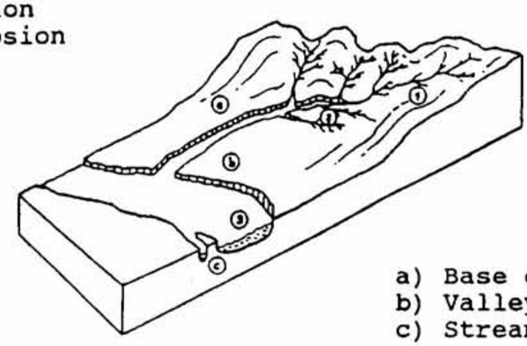

Fig. 1: Erosion-Sedimentation process; (a) Rainfall storm.
(b) Detachment by rain-splash and overland flow.
(c) Transportation and Sedimentation along the stream.

Due to rainfall energy and rain-splash, the soil particles are detached from the soil surface and made ready for transportation. The overland flow has the effect of detaching more particles and transporting them to the stream. Detachment by flow can occur when transport capacity exceeds sediment load. A detachment capacity can be defined on the local conditions but the actual detachment rate depends on the degree to which the transport capacity is filled. Part of the detached particles are deposited on the deposition occurrences along the way (Fig. 2). The three processes, detachment, transport and deposition, may also occur in the channel itself. This depends on the physical characteristics of the discharge and its ability to hold the Soil particles.

Sediment Yield from watershed results when the upland erosion exceeds deposition (Onstand and Foster, 1975). The major factors affecting sediment yield of a drainage area are (ICOLD, 1989):

Erosion

1) Sheet and rill erosion
2) Gully erosion
3) Channel erosion



Deposition

a) Base of steep slopes.
b) Valley deposition.
c) Streambed aggradation.

Fig. 2: Typical erosion and deposition occurrences (Foster, 1970)

a. Rainfall amount and intensity.
b. Soil type and characteristics.
c. Ground cover (vegetation, litter and rock fragments).
d. Land use (cultivation practices, grazing, logging, construction activities and conservation practices).
e. Topography.
f. Erosion history (nature of drainage network, density, slope, size and alignment of channels).
g. Runoff.

**The Universal Soil loss Equation**

The Universal Soil Loss Equation (USLE) after Wischmeier and Smith was developed for predicating average annual sediment Yield from cropland. The above mentioned factors are considered in the parameters of the USLE. This equation has the following form (Wischmeier and Smith, 1960):

$$A = R.K.L.S.C.P \qquad ....(1)$$

where

A: soil loss (kg/ha/year)
R: rainfall erosivity factor
K: soil erodibility factor (kg.h/(ha.N))
L: slope Length factor
S: slope steepness factor
C: cover management factor
P: support practice factor

The first parameter of the USLE represent the energy of the erosive agent, the rainfall. The other parameters represent the characteristics of the watershed and its soil cover and are to be taken from prepared tables and diagrams (Wischmeier and Smith, 1978).

The rainfall erosivity factor R is determined by considering the energy of all the storms occurring during the year. For a single storm, the erosivity factor R is calculated using the empirical equation after Wischmeier and Smith (1958):

$$R = (11,89 + 8,73 \cdot I_i) \cdot I_{30} \qquad \text{....(2)}$$

where

R : rainfall erosivity factor (N/h)
$I_i$ : rainfall intensity (mm/h)
$I_{30}$: maximum 30-minutes intensity (mm/h)

To estimate the sediment yield of a single storm, the USLE was modified several times. The modification after Auerswald (1990) has the following form:

$$Y = R.K.L.S.RD.P \qquad \text{....(3)}$$

where

Y : sediment yield from an individual storm (kg/ha)
R : rainfall erosivity factor of the specified storm (N/h)
RD: relative detachment
K, L, S and P are the same as in the USLE

The relative detachment RD gives how much soil is available for erosion relative to the standard soil which was used by the USLE. This factor is given in prepared tables as a function of the soil type and vegetation cover (Schwertmann et al. 1987).

**The sediment continuity equation**

The basic governing equation of the erosion process on upland areas is the equation of continuity for sediment transport by overland flow (Bennett, 1974). In the following analysis, negligible dispersion within the flow and a quasi-steady flow are assumed which simplifies the continuity equation to (Foster and Meyer, 1972):

$$\frac{ds}{dx} = D_r + D_i \qquad \ldots\ldots(4)$$

where

$Q_s$: sediment load (kg/(m.s))
$D_r$: detachment rate by rill erosion (kg/($m^2$.s))
$D_i$: detachment rate by interrill erosion (kg/($m^2$.s))

The two detachment rates occurs simultaneously and are very difficult to be differentiated. Therefore a further simplification of the continuity equation by combining both detachment rates in one function is acceptable (Shaheen, 1992). Thus the above equation can be written as:

$$\frac{dQ_s}{dx} = D \qquad \ldots.(5)$$

where D is a detachment rate that gives the detachment in kg/($m^2$.s) from the whole watershed.

**The Exponential Detachment equation.**

One of the most popular soil detachment equations is the exponential detachment equation which was implemented by many models (SWWM 1971, STORM 1976 ILUUDAS 1979 KOSIM 1987). A simple form of the exponential equation is (williams 1978):

$$D(t) = D_o \cdot e^{-bt} \qquad \ldots.(6)$$

where

D(t): detachment at time t (kg/(ha.h))
$D_o$ : Detachment at the beginning of the storm (kg/(ha.h))
b : detachment coefficient (1/h)
t : time

In the developed model the continuity equation in its simple form (Eq. 5) along with the exponential detachment equation (Eq. 6) are integrated over space and time using the convolution integral and the time-area method.

**Mathematical representation of the Model**

In this model the sediment hydrograph is predicted by convolution of the continuity equation in its simple form (Eq. 5) over space and time. For the detachment rate the exponential detachment function is used. Further assumptions is that sediment is distributed uniformly over area and time and that linear superposition dose apply for the detachment process. Therefore the model can be mathematically written as:

For $t \leqslant T_c$

$$Q_s(t) = \int_{\tau=0}^{0<t<T_c} [F(\tau+\Delta t) - F(\tau)] \cdot D_o \cdot e^{-b(t-\tau)} \, d\tau \qquad \ldots.(7)$$

For $t>T_c$

$$Q_s(t) = Q_s(T_c) \cdot e^{-b(t-Tc)} \qquad \ldots.(8)$$

and

$$F(t) = \frac{A}{T_c^{\,2}} t^2 \qquad \ldots.(9)$$

where

$Q_s$: Sediment load at time t (kg)
$D_0$: potential Detachment (kg/(ha.h))
b : Detachment coefficient (1/h)
$T_c$: Time of concentration of sediment wave (h)
A : Area of the watershed (ha)
t : Time since beginning of the storm (h)
$\tau$ : dummy time variable (h)

The two model parameters $D_o$ and b are to be calibrated using measured data, where $D_o$ is function of the erosivity factor R and $D_o/b$, which represent the thickness of the detachable soil layer, is function of the sediment yield of the storm (Shaheen 1992):

$$b = a \cdot R^m \qquad \ldots.(10)$$

$$D_o/b = c \cdot Y \qquad \ldots.(11)$$

Where a, m and c are regression factors.

The idea of the model is clarified by Fig. 3 (a), in which the principle of time-area diagram is used and the linear superposition is applied.

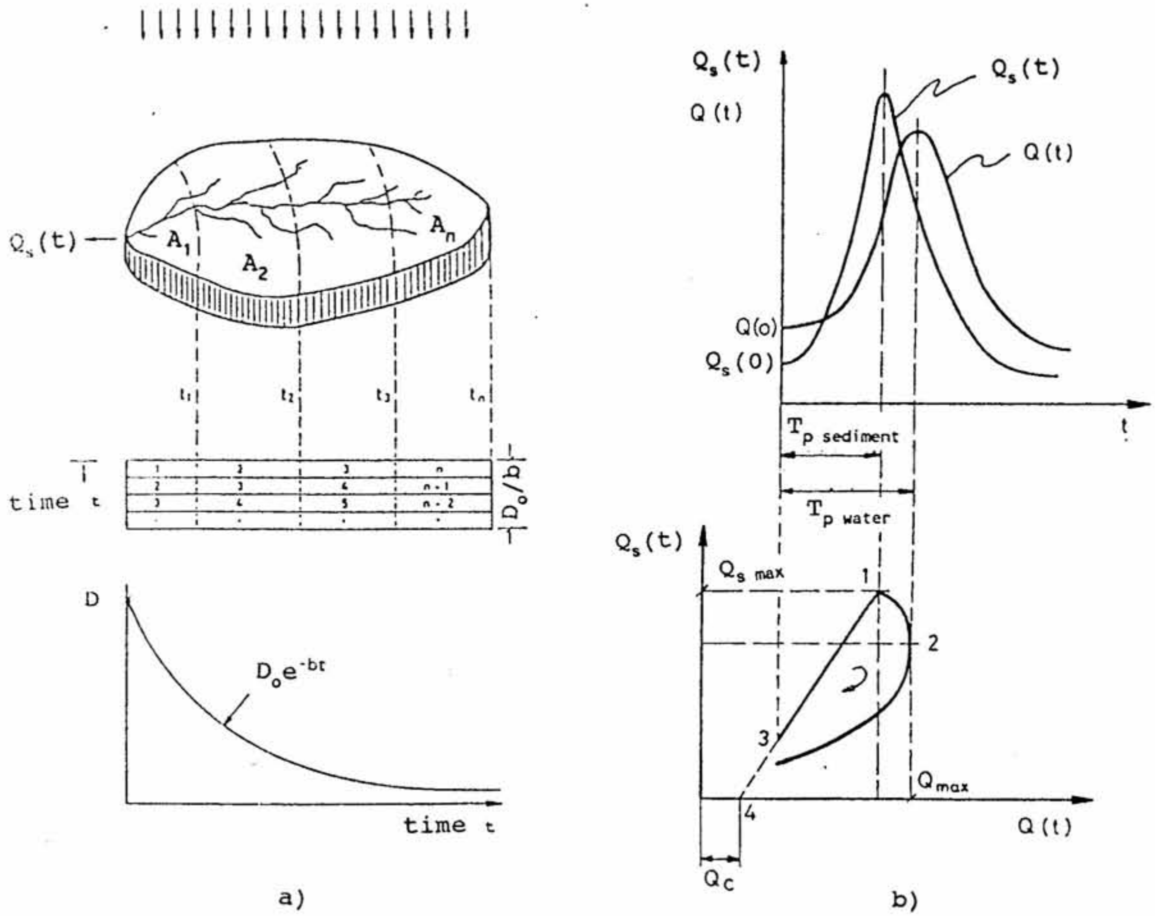



Fig. 3: (a) A sketch representing the idea of the model. (b) The sediment loop and its relation to sediment and water hydrographs (Shaheen, 1992)

A base sediment flow is introduced to be added constantly over the base time of the sediment hydrograph and is to equal the initial of the sediment flow, which is a function of the critical shear stress required to initiate the sediment movement and is given by (Foster, 1982):

$$Q_s(0) = a_s \cdot (\tau_o(0) - \tau_c)^{1,5} \qquad \ldots\ldots(12)$$

where

$Q_s(0)$: base sediment discharge (kg/s)
$a_s$ : soil factor $((m^{1,5} \cdot s^2)/kg^{0,5})$
$\tau_o(0)$: shear stress at time t=0 $(N/m^2)$
$\tau_c$ : critical shear stress $(N/m^2)$

One of the most important results of the presented model is the sediment loop (Fig. 3 (b)) with the following notations:

1. The loop consist of two parts; a straight line from the base sediment flow to the maximum flow (point 1) and a curve which represent the falling limb of the sediment hydrograph.

2. Time of concentration of sediment hydrograph is less than that of the water hydrograph. This goes well with the fact that the detachment begins early and that high sediment concentration are measured at the beginning of the storm. In the application of the model the sediment time of concentration was taken to be 0,95 of the water time of concentration.

3. The intercept of the straight line with the discharge axis (point 4) gives the critical discharge which is required to start the transportation of the sediment particles.

**Results and discussion**

The applicability and limitations of the model is best demonstrated by applying the method to a drainage basin that satisfies the data requirements outlined above. Observed sediment graphs on a watershed in the Federal Republic of Germany were compared with the corresponding sediment graphs obtainable from the model.

Glonn watershed has an area of 47 hectare and is 521 to 550 m above sea level (Fig 4). The measurements in the watershed were taken by Bavarian Office for Water Management in Munich during the three years 1980 to 1982. Nine storms of these measurements were tested using this model. The information about these nine storms are listed in Table (1).

The estimated sediment yield for each storm resulted from applying the model after Auerswald (Eq. 3) along with the different parameters of the equation are listed in Table (2). In the table the two factors; slope length L and slope steepness S are combined in one factor LS called topography factor.

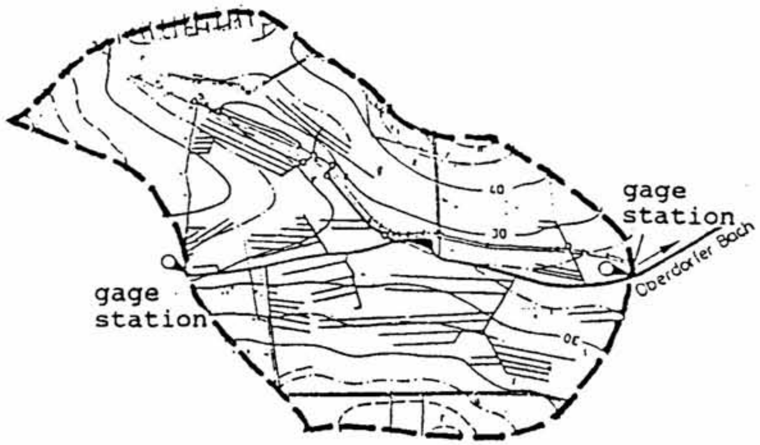



Fig. 4: Sketch map of Glonn watershed

Table 1: Relatively isolated storm events in Glonn watershed

| event date | Rainfall total min | Rainfall effec. mm | peak discharge l/s | sediment yield kg | dry period h |
|---|---|---|---|---|---|
| 06.11.79 | 20,6 | 3,82 | 72,07 | 4200 | |
| 08.11.79 | 7,9 | 0,35 | 44,57 | 2273 | 24 |
| 04.02.80 | 13,1 | 1,39 | 86,0 | 6996 | 271 |
| 25.04.80 | 17,6 | 2,74 | 104,13 | 3338 | 387 |
| 07.06.80 | 23,9 | 5,18 | 340,94 | 16731 | 206 |
| 10.07.80 | 7,8 | 0,35 | 36,86 | 658 | 21 |
| 20.07.80 | 15,4 | 2,03 | 35,7 | 278 | 105 |
| 30.11.81 | 17,2 | 2,60 | 59,00 | 847 | 41 |
| 29.06.82 | 18,8 | 3,15 | 29,18 | 1183 | 63 |

The comparison of the measured sediment graphs with the computed sediment graphs is illustrated in Fig. 5. The convolved sediment graphs in each case are noted to be good approximation to the actual storm sediment graphs. Differences between the two sediment graphs are seen that large relative errors occur only at both ends of the sediment graph. Apart from this, the model generates a closely comparable sediment graph to the naturally observed one.

Table 2: Results of applying Auerswald model (Eq. 3) to the events of Glonn watershed

| event date | $R.10^3$ N/h | RD - | K t·h/(ha·N) | LS - | P - | Y kg/ha |
|---|---|---|---|---|---|---|
| 06.11.79 | 732 | 0,258 | 0,36 | 2,1 | 1 | 142,8 |
| 08.11.79 | 281 | 0,258 | 0,36 | 2,1 | 1 | 54,3 |
| 04.02.80 | 384 | 0,258 | 0,36 | 2,1 | 1 | 74,2 |
| 25.04.80 | 417 | 0,492 | 0,36 | 2,1 | 1 | 155,1 |
| 07.06.80 | 3884 | 0,131 | 0,36 | 2,1 | 1 | 384,7 |
| 10.07.80 | 293 | 0,055 | 0,36 | 2,1 | 1 | 12,2 |
| 20.07.80 | 591 | 0,046 | 0,36 | 2,1 | 1 | 20,6 |
| 30.11.81 | 219 | 0,258 | 0,36 | 2,1 | 1 | 42,7 |
| 29.06.82 | 1280 | 0,099 | 0,36 | 2,1 | 1 | 95,8 |

After calibrating the model parameter b and $D_o$, using the data of the above described watershed, the three regression factors a, m, and c (Eqs. 10 and 11) was found to be 0,67; 1,19 and 0,75 respectively with a regression coefficient of about 0,9.

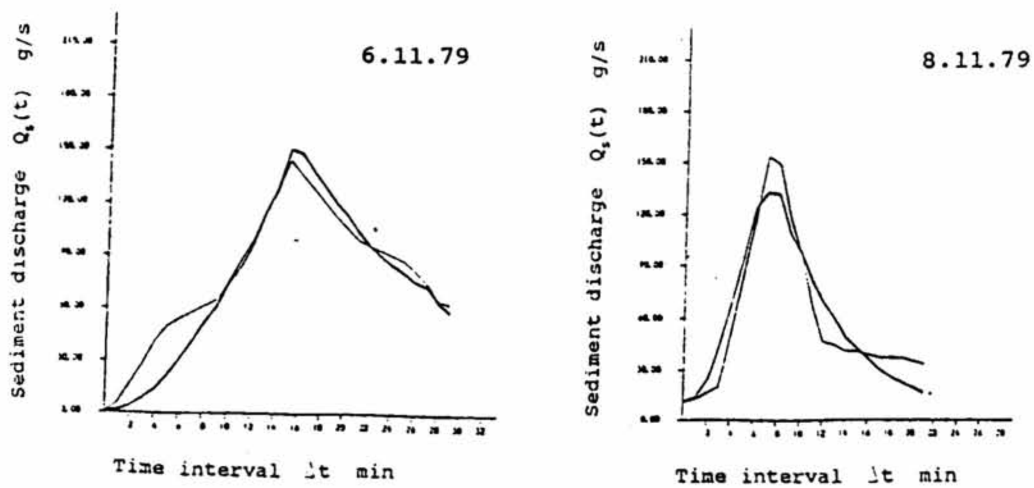



Fig. 6: Comparison of measured and estimated graphs for the storms of Glonn watershed

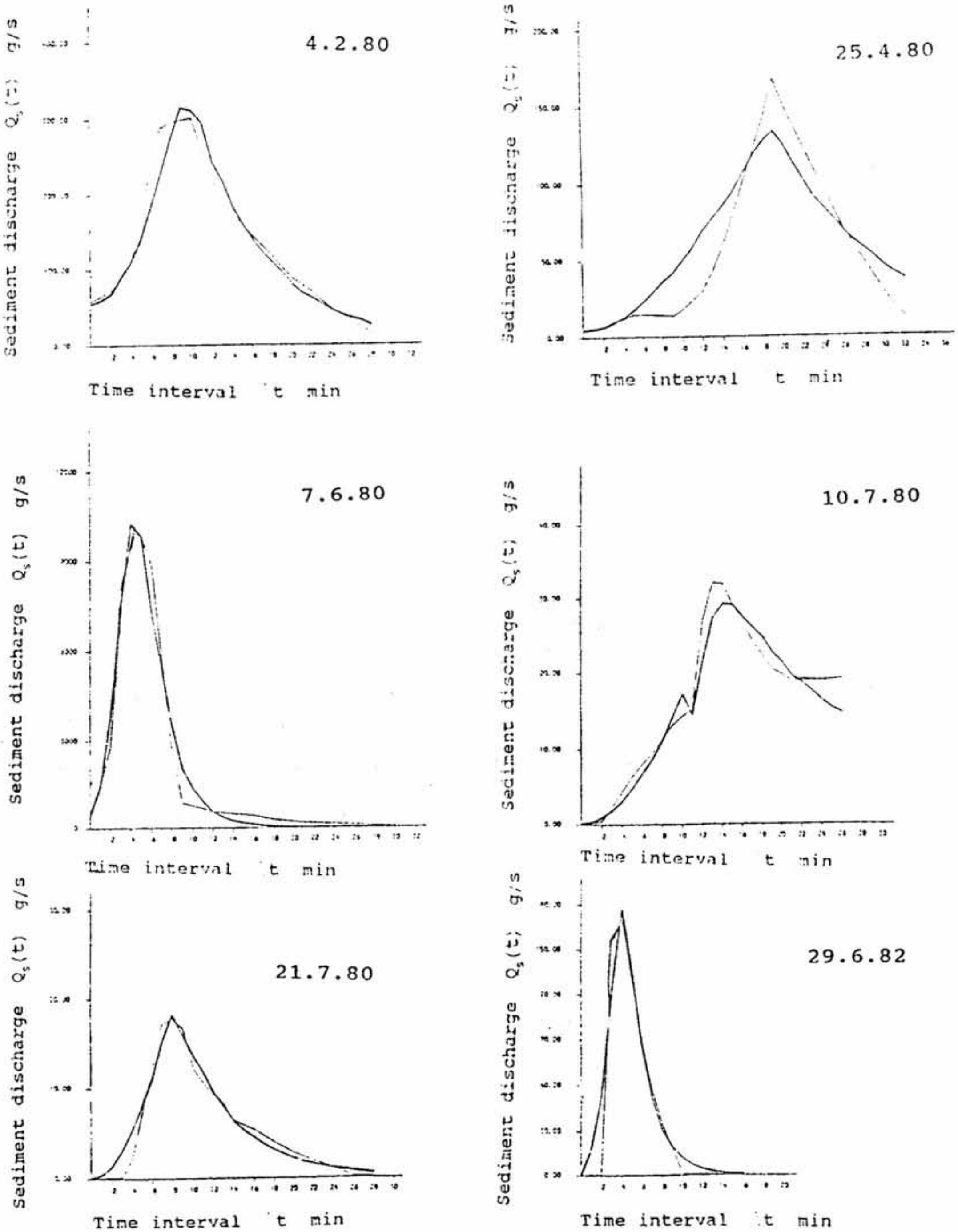



Fig. 6: Comparison of measured and estimated graphs for the storms of Glonn watershed (continue)

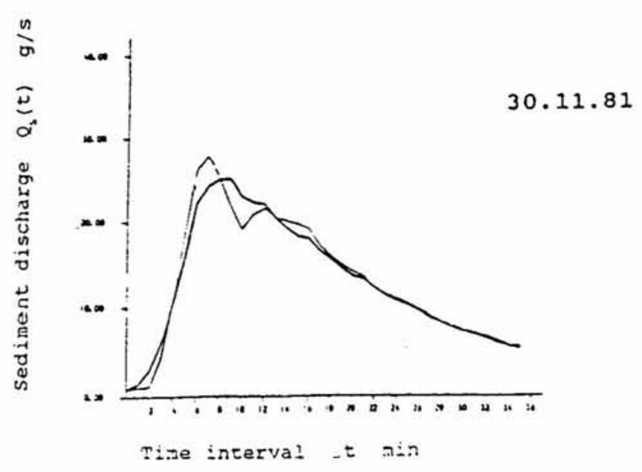



Fig. 6: Comparison of measured and estimated graphs for the storms of Glonn watershed (continue)

**Conclusions**

The proposed method to model sediment graphs resulting in a watershed subjected to a rainfall storm has limitations as well as providing insight into the dynamics of sediment detachment and transport. The results have shown the adequacy of the linear reservoir concept for the functional representation of a natural watershed. The close agreement between the computed sediment graphs and the observed graphs is very favorable. By means of this simple method of calculation an accuracy is attained that more than satisfies the practical requirements.

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