Persistent Currents in Normal Metal Rings:
Old Questions, New Answers

Ulrich Eckern (a) and Peter Schwabb (b)
Institute of Physics, University of Augsburg, 86135 Augsburg, Germany
a) e-mail: ulrich.eckern@physik.uni-augsburg.de
b) e-mail: peter.schwab@physik.uni-augsburg.de

Abstract:

The Aharonov-Bohm effect, first described in 1959, is among the most spectacular effects of quantum mechanics, emphasizing the role the electromagnetic potentials – and not the electromagnetic fields – play for the wave-like motion of quantum particles. Considering a ringlike geometry in a constant perpendicular magnetic field, a direct consequence is that all properties of a charged system are periodic functions of the magnetic flux, $\Phi$, the flux periodicity given by the fundamental flux quantum, $\Phi_0 = h/e$. This result is based on the particular combination, $p + eA$, which appears in the Hamiltonian of the system, where $p$ is the momentum, and $A$ the vector potential; here we consider electronic systems, and the charge of an electron is $-e$.

In equilibrium, the system’s properties can be calculated from the partition function, which involves a trace over all states of the systems: hence in the classical limit any flux dependence disappears (Bohr-van-Leeuwen-Theorem), and the persistent current, $I(\Phi) = -\frac{\partial F(\Phi)}{\partial \Phi}$, vanishes; $F(\Phi)$ denotes the thermodynamic potential. Thus very small systems and very low temperatures are required for a finite (non-zero) $I(\Phi)$ to exist.

In fact “normal” persistent currents, of the order of a few nA, have been seen in several experiments, for temperatures below 1 K [1–4].
In contrast to the experiments [1–3] which used a SQUID technique in order to detect the magnetic moment induced by the current, the most recent study [4] employed a nano-electromechanical technique: the rings were placed on a cantilever, whose oscillation frequency can be measured with extremely high accuracy. The perimeter of the studied rings varied between 0.6 and 1.6 μm.

Assuming time reversal invariance, the Fourier expansion of the persistent current is given by

\[ I(\Phi) = I_1 \sin \left( \frac{2\pi \Phi}{\Phi_0} \right) + I_2 \sin \left( \frac{4\pi \Phi}{\Phi_0} \right) + \ldots. \]

Because of the disorder always present in small rings, due to the fabrication process, the amplitudes I1 and I2 are random quantities. For example, I1 fluctuates from ring to ring; in particular, it changes its sign, hence the average is expected (and was found) to be zero. The size of \( \langle I_1^2 \rangle^{1/2} \), on the other hand, has been debated for many years; the theoretical prediction \( \langle I_1^2 \rangle^{1/2} \approx E_c/\Phi_0 \) [5] was now convincingly confirmed [4]; here \( E_c = hD/L^2 \) is called Thouless energy, D is the diffusion constant, and L the perimeter of the ring. Concerning the second harmonic, I2, it was pointed out rather early [6] that the effective electron-electron interaction gives an important contribution, which nevertheless is too small compared to the experimental result [1]. A recent paper discusses the question whether a small amount of paramagnetic impurities can resolve this discrepancy [7]. For an introduction into persistent currents, see [8].