

## Critical points at infinity in the variational calculus: An overview

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Abstract:

The standard contact structure  $\alpha_0$  of  $S^3$  has a vector-field  $v$  defining a Hopf fibration in its kernel. Legendre transform w.r.t  $v$  can be performed. Symmetric Hamiltonian problems are thereby transformed into their Lagrangian counterparts. It was believed that the existence of such a  $v$  was special to this framework. This belief turns out to be wrong. V. Martino has produced a vector-field in the kernel of the first contact form  $\alpha$  by J.Gonzalo and F.Varela such that  $d\alpha(v, \cdot)$  is also a contact form with the same orientation than  $\alpha$ . This provides a new textbook example in Contact Form Geometry. We will describe in our talk the first contact form of J.Gonzalo and F.Varela and the vector-field  $v$  in its kernel by V.Martino; we will study the related dynamics and the related Reeb vector-fields periodic orbit problems at the light of the homology for contact forms/structures that we have defined in our work.