An-Najah National University

Faculty of Graduates Studies

Simplified Conceptual Equation for Soil-Structure Interaction for Simple Structures Due To Vertical Loads for Practical Purposes

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Dedication

Praise be to Allah, Lord of the worlds

To the Prophet Mohammad
Blessings and Peace be upon him

To my father
To my mother
To my precious ones
To all friends and colleagues
To my teachers

To everyone working in this field
To all of them

I literally dedicate this work

Acknowledgment

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For their guidance, suggestions and assistance during the preparation of this thesis

الإقرار

أنا الموقع أدناه مقدم الرسالة التي تحمل العنوان

Simplified Conceptual Equation for Soil-Structure Interaction for Simple Structures Due To Vertical Loads for Practical Purposes

أقر بأن ما شملت عليه الرسالة هو نتاج جهدي الخاص, باستثناء ما تمت الإشارة إليه حيثما ورد, وأن هذه الرسالة ككل أو أي جزء منها لم يقدم من قبل لنيل أي درجة أو لقب علمي أو بحثي لدى أي مؤسسة علمية أو بحثية

Declaration

The work provided in this thesis, unless otherwise referenced, is the researcher's own work, and has not been submitted elsewhere for any other degrees or qualifications.

Student's Name: **اسم الطالب: فوزي أبو العدس**

Signature **التوقيع:** فوزي أبو العدس

Date **التاريخ: 1/ 11 /2016**

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Abstract

The non-uniform settlements represent a big challenge for the structural engineers due to the problems caused by this phenomenon. Many cracks in the walls, columns and slabs occur due to such non-uniform settlements. These cracks range from small cracks to major cracks that may threat the safety of the building and the residents.

Along the years, geotechnical engineers have developed many methods to find settlements in soil. However, these methods need certain expertise and knowledge in the properties and the conditions of soil, which many structural engineers are poor at. Therefore, and because of the importance of the soil-structure interaction, this study focuses on proposing simplified equation to estimate the settlements of soil with acceptable accuracy for practical purposes, like design or field checks.

To simplify the process, the displacement will be presented as ratios and will be used as a reference for the fitted equation, where the soil settlement and the displacement of structure are assumed dependent by taking the ratios to the total displacement, which represents the summation of the soil settlement and the displacement of structure. By knowing displacement of

structure and the displacement ratio, the soil settlement can be found, and vice versa.

Within this thesis, the applicability of the main assumptions used in soil structure interaction will be demonstrated for simple structure of one square column and footing and simple two-span frame with identical columns and footings dimensions.

The finite elements method will be used as the calculation tool for the displacements of the structure and the soil, where to assure acceptable accuracy, the soil and the structure will be simulated as multi nodded three-dimensional elements, meshed to certain dimensions that give accurate results.

The equations for the displacement ratios will be fitted using the finite elements results, and the results will be discussed by conducting comparisons between the results from finite elements and the equations, in order to assess the accuracy of the equations' results.

1 Introduction and literature review

1.1 Introduction

Structural design and construction have achieved significant breakthroughs during the last fifty years, and the engineering methods have improved significantly since the existence of the technological tools and the development of the finite elements methods, which enable the designers to produce optimal and economical structures and more accurate results that approach the real behavior of the structures.

In the beginning of the twentieth century the structural designers have concluded an assumption to consider the structure as a flexible object, while considering the soil as a rigid body, which is commonly represented as totally or partially restrained joints with respect to all directions. On the other hand, the geotechnical engineers have an opposite assumption of considering the soil as a flexible object, while considering the structure as a rigid object. (Lai, Martinelli 2013).

These practices are considered acceptable among both structural and geotechnical engineers, and were applied in the structural design for both vertical and horizontal forces, and are still used widely among the structural design firms in Palestine because of its simplicity. Despite of this acceptance, these methods were proved to be inaccurate and have significant errors that cause severe damages to structural members, which are considered safely designed and free of risk according to these proposed assumptions, especially when the soil is classified as soft soil. Therefore, both the soil and the structure must be considered flexible, where realistic model of soil structure interaction can lead to an optimal and economical structure (Breeveld, 2013).

Therefore, many methods have been developed to find the settlement of the soil in order to study its effect on the structural members. These methods are considered common within the geotechnical engineers. However, they are considered difficult to perform for the common structural designers, because of the need of certain geotechnical expertise, specific parameters and data.

Thus, the objective of this thesis is to fit a simplified equation that can predict the settlement of soil easily, to simplify the structural design process and other practical purposes like field checks and failure analyses.

1.2 Definition of soil-structure interaction

(Kausel, 2010) defines the soil-structure interaction as “an interdisciplinary field of endeavor which lies at the intersection of soil and structural mechanics for both static and dynamic behaviors”. The soil-structure interaction represents the link between the earthquake engineering, geophysics and geomechanics, mechanics of materials and computational and numerical methods.

Thus, the soil-structure interaction is the practice that includes the structural and the geotechnical properties in the analysis process, in order to figure out the effects of specific forces on the whole system; i.e. to study the true behavior of the structure and soil. Understanding the effect of the interaction between these two major objects is essential to predict the reactions in the design process, to introduce safer and economical structural forms.

1.3 Soil structure interaction components

After defining the soil-structure interaction, the components that the soil-structure interaction system consists of will be defined as:

1.3.1 Structure

(Hibbeler, 2009) simply defines the structure as “a system of connected parts, which used to support a load”. Another detailed definition concluded that the structure is a system of connected elements in a stable condition that has the ability to support external loads and resist external pressures and internal stresses without failing.

Plenty of materials are used for structural purposes, some structures are built using single material like steel and wood, and some are built with a combination of materials that support each other, like reinforced concrete. The used structural system depends on the materials available in the area of building. Therefore, almost all the structures in Palestine are built using reinforced concrete.

The structure consists of many structural members, and each has its specific function. Columns are the members that resist the axial loads. Beams resist the bending moments and shear forces. Slabs are the functional members of the structure that support the vertical loads. The foundations are the members that distribute the loads into the soil.

The foundations can be sorted into: single footings, which are the foundations that support one column. The combined footings and strap footings, which support two or more columns. The wall footings, which support the structural walls, either bearing walls, shear walls or retaining walls. And there are the raft footings, which support group of structural elements over large areas. The previously mentioned types are considered shallow foundations, where the footings are located approximately near the surface. In addition, pile and pier foundations, which are deep foundations, support the structure when the soil is very weak or the structure is very heavy. Piles can have lengths that reach hundreds of meters into the soil in the heavy important structures.

1.3.2 Soil

For engineering purposes, (Das, 2013) defines the soil as “the uncemented aggregate of mineral grains and decayed solid organic matter with liquid and gas in the empty spaces between the solid particles”. Soil is usually used as a building material in many of civil engineering practices. The origin of the Portland cement used in the reinforced concrete is soil, specifically Limestone, clay and ashes (Nilson, Darwin and Dolan, 2010). Moreover, it is the part that supports the foundation of the structure, therefore supporting the whole structure. Soil classification systems divide soils into groups and subgroups based on various engineering criteria, such as grain size, liquid limit, and plastic limit. (Das, 2011).

1.3.1.1 Importance of soil study for structural engineering

Studying the soil has high significance in structural engineering, because it is the part that supports the structure, and any miscalculation is dangerous and costly. Therefore, the soil condition must be studied carefully in order to achieve high safety factor, and to avoid any failures that may occur in the structural members due to failure in soil. (Verruijt and Van Baars, 2007) clarified why studying the soil is important in the following points:

- The soil stiffness depends on the stress level: In general, materials like steel, wood or even concrete have linear stress strain behavior up to a certain level, which means if the stresses doubled; the strains will be twice as large, assuming the stresses are in the elastic range. On the other hand, the stiffness of the soil increases by increasing the compression stresses affecting the soil particles. This is mainly caused due to the increase of the forces between the individual particles when the external compression stresses increased, which gives the structure of particles more strength, thus more stiffness.
- Shear: In contrast of the previous point, soil becomes gradually softer in shear, and if the shear stresses reach a certain level, with respect to the normal stresses, the possibility of having a failure in the soil mass increases. The reason is that the soil particles will slide over each other with greater slopes, which will lead to failures.
- Dilatancy: A phenomenon discovered by Reynolds in 1885, which is related to the change of the soil volume. To clarify this phenomenon, a simple example of loose saturated soil affected by significant

pressure is presented. The excessive stress will cause shear failure in the soil, decreases the soil volume and reduces the water pores of the soil, and the water will turn to the nearby area causing volume expansion and liquefaction of soil. This behavior is very dangerous for the offshore structures, and can cause significant failures.

- Creep: The deformation of soil depends on time. Therefore, the duration of deformation depends on the soil classification and the pores between the soil particles. For example, the settlement of sand and hard aggregates will finish after short time, while the settlement of clay will last for longer time that may reach years.
- Ground water: It is from the soil characteristics to have water pores between the soil particles; this water can affect the soil mechanical properties and stress resistance by changing the friction between the particles and increasing the settlement of the soil. Usually, two cases of the soil are studied in every soil sample, the saturation phase, where the soil contains water pores, and the dry phase, where the soil is dry and no water within it.
- Non-uniform initial stresses: The initial stresses that affect the soil are often not uniform and even partly unknown or hard to determine, because of the non-homogeneity nature of the soil. However, because of the soil non-elastic behavior mentioned earlier, it is important to have an idea of the initial stresses to take into consideration when designing the structure. The vertical stresses can be approximately predicted from the weight of the soil by predicting

the soil density. On the other hand, the horizontal stresses remain unknown, and hard to be predicted.

- Variability: The probability of having different soil properties on different locations is high due to the creation of soil by ancient geological processes. Even in two very close locations the soil properties may be completely different. In addition, the soil is usually deposited as multi layers, with various thicknesses and properties. Not knowing the properties and the thicknesses of the layers affected by the stresses may cause significant failures, especially for the heavy-weighted important structures.

1.4 Soil-structure interaction simulation types

From the definition of the soil-structure interaction, it is obvious that any model should contain the two parts of the system, the structure and the soil. Thus, many modeling systems were developed to solve the problem of representing the soil in the structural system. Two main methods are the most common to be used in the soil-structure interaction simulations; the direct approach and the substructure approach.

1.4.1 Direct approach

The direct approach depends on the actual presentation of the soil volume as a structural object in addition to the super structural members and the whole system is analyzed as one unit using one of the finite elements methods, (Lai, Martinelli 2013). In three dimensional modeling this approach usually demands the modeling of the soil as a three dimensional solid member with a specific depth and properties and is attached to the three dimensional super structural elements and the sub structural elements, thus they act as one unit. Figure 1.1 shows a direct approach model, and shows the parts that the direct approach consists of.

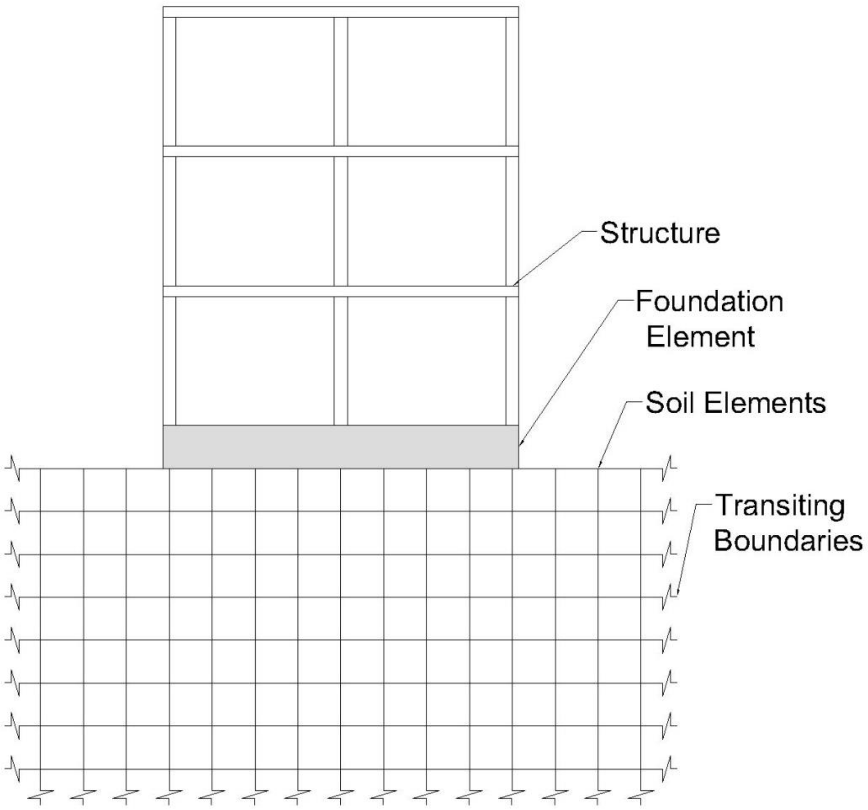

Figure 1.1: Typical direct approach model.

1.4.2 Substructure approach

The substructure approach, or the indirect approach in some references, uses an equivalent object with certain properties to replace the volume of soil. As defined by (Kausel and Roesset, 1974) the substructure approach is a “technique by which a soil-structure interaction problem is solved by decomposing the superstructure-foundation-soil system into two subsystems, whose response is determined independently”. The total response of the overall system is then obtained from the application of the theory of superposition. Commonly a set of springs, dampers and other structural objects are used to form a behavior which is close to the soil's behavior under dynamic stresses like earthquakes.

Figure 1.2 shows the substructure model, and the simple parts used to simulate the soil. This approach is considered somehow easy to use and not time consuming. Thus, it is the preferred approach to be used for design purposes for usual structures. Although, it has many disadvantages, such as the need for some geotechnical theory for equivalent soil simulation. Also, because it is an indirect approach and depends on an equivalent model and assumed conditions, the results must have a certain percentage of error.

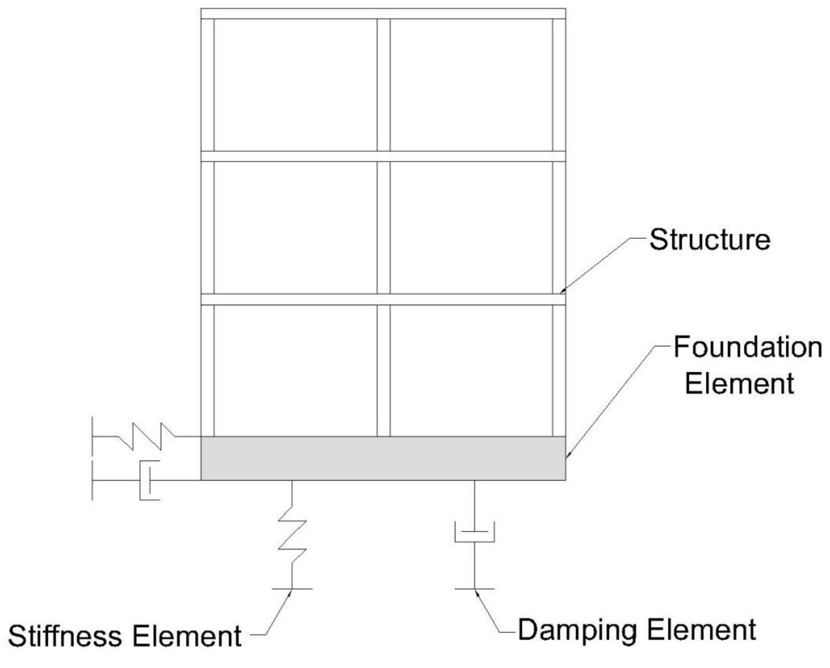

Figure 1.2: Typical substructure approach model.

1.5 Importance of soil-structure interaction

Underestimating the soil-structure interaction effects may cause structural problems, which sometimes cause severe damages for the structural elements due to the unexpected soil settlements. For example, consider a structure built on two types of soil with significant difference in stiffness. Ignoring the soil displacements will lead to non-uniform settlement that will cause unexpected stresses that the structural members may not sustain, which leads to failure. Figure1.3 shows a sketch for a two-span frame with non-uniform settlement in the middle footing, where the expected cracks are shown in the figure.

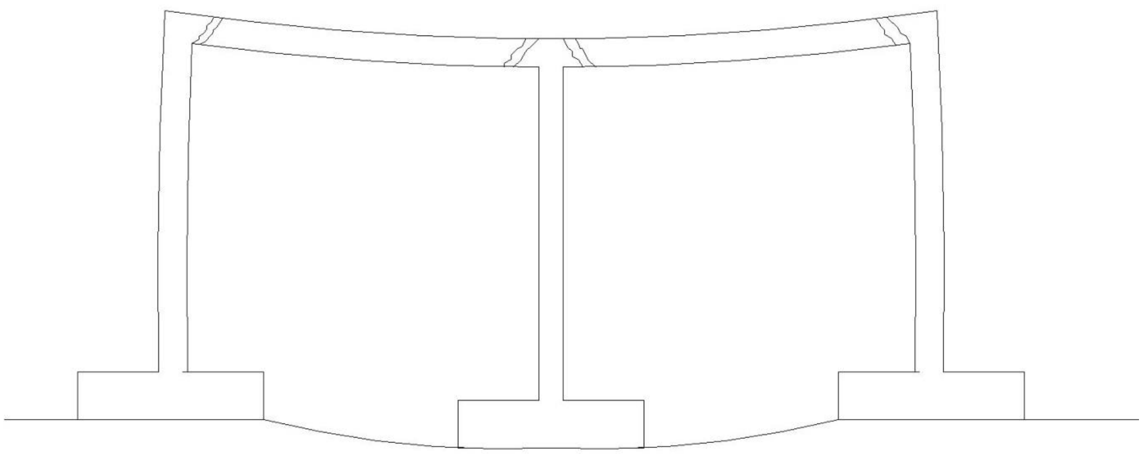

Figure 1.3: Two-span frame with non-uniform settlement in the middle footing.

While the settlement difference between the footings increases, the stress affecting the structural members changes significantly, where for certain settlement the beam acts like one span beam reversing the expected stresses of the middle joint, and increasing the stresses at the edge joints significantly.

Figure 1.4 shows moment diagram for two-span frame with fixed soil, while Figure 1.5 shows moment diagram for the same frame with flexible soil, assuming significant stiffness difference for the middle soil by reducing the modulus of elasticity. The differences between Figure 1.4 and Figure 1.5 are very obvious, where for the middle joint, the tension and compression forces are reversed, which means the negative reinforcing steel is useless, and the bottom part of concrete is affected by tension forces. No such stresses were taken into consideration for the design process, and the cracks in concrete depend on the magnitude of the tension forces. On the other hand, the negative reinforcing steel for the edge joints are approximately doubled, which will cause significant failure if the tension stresses exceed the reinforcing steel capacity.

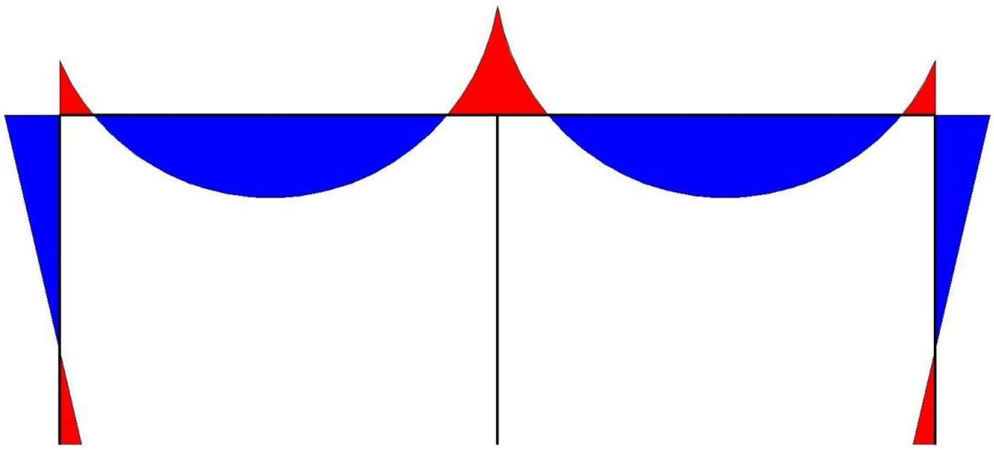

Figure 1.4: General sketch of bending moment diagram for two spans frame with fixed soil.

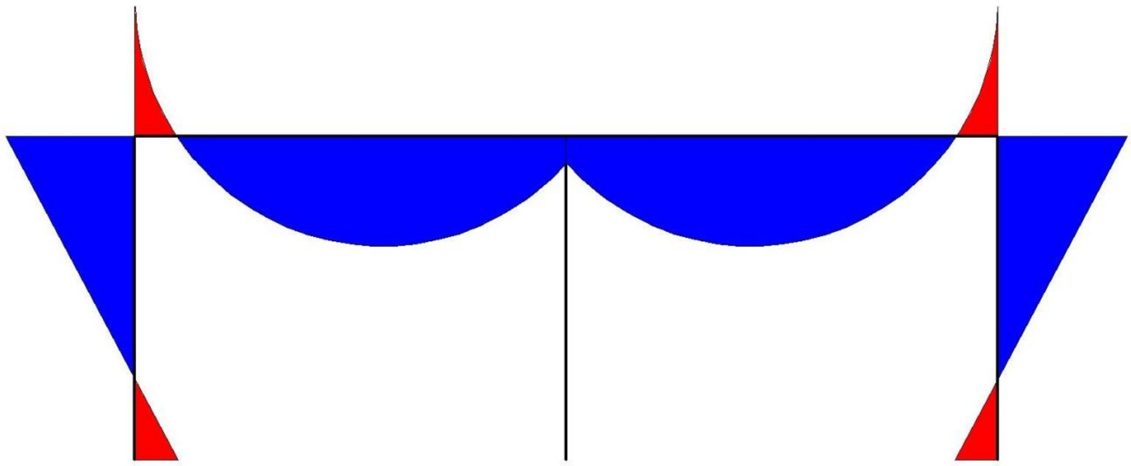

Figure 1.5: General sketch of bending moment diagram for two spans frame with large settlement in the middle column.

1.6 Soil settlement calculation methods

(Das, 2009) states that, in general; settlement of a foundation consists of two major components, elastic settlement and consolidation settlement. In the granular soils, the elastic settlement is the predominant settlement. On the other hand, in the saturated inorganic clay and silts the predominant settlement is the primary consolidation (Das, 2008).

(Das, 2009) sorts the settlement calculation methods into three main categories depending on the methodology, which are:

1. Methods based on observed settlement of structures and full scale prototypes. These methods are empirical, and depend on the results from empirical tests, like standard penetration test (SPT) and the cone penetration test (CPT). Many methods are developed to find the settlement empirically: Terzaghi and Peck (1948, 1967), Meyerhof

(1965), DeBeer and Martens (1957), Hough (1969), Peck and Bazaraa (1969), and Burland and Burbidge (1985).

2. Semi empirical methods. These methods are based on a combination of field observations and some theoretical studies. They include the procedures outlined by Schmertmann (1970), Briaud (2007), and Akbas and Kulhawy (2009).
3. Methods based on theoretical relationships derived from the theory of elasticity. The relationships for settlement calculations available in this category contain the term modulus of elasticity of soil.

Many generalized methods were developed to find the average immediate soil settlement depending on the theory of elasticity like (Janbu et al. 1956), which have been improved by (Christain and Carrier, 1978).

(Mayne and Poulos, 1999), presented an improved equation for elastic settlement, where the rigidity of the foundation, the increase of modulus of elasticity of the soil depth, the embedment depth of foundation and the rigid layers location at a limited depth (Das, 2008).

1.7 Problem statement

As mentioned in the previous section, there are many methods to calculate the soil settlements. However, these methods have many disadvantages that do not encourage the structural engineers to use them. Categories 1 and 2 are based on assumptions for standard situations, thus they need specific charts, tables, unit conversion and constant correlation, in order to get acceptable accuracy. Category 3 is considered general because it is an analytical method and is based on mechanics of materials and soil mechanics theories and properties. However, it is complicated, time consuming and needs certain expertise in mathematics and finite elements tools and soil properties.

1.8 Research objectives

The main objective of this research is to obtain a simplified equation that predicts the settlement of soil due to vertical loads, in order to help the structural engineers to take the soil structure interaction into consideration in a practical way. Because shallow single footings are the most common type used in the country, the research will study the soil-structure interaction behavior for this type only.

To simplify the calculations for the structural engineers, an assumption is made to relate the soil settlement to the displacement of structure by finding out the ratio of these displacements to the total displacement: soil settlement to total displacement $\frac{\Delta_{soil}}{\Delta_{total}}$, and displacement of structure to total displacement $\frac{\Delta_{structure}}{\Delta_{total}}$, where the total displacement is the sum of the displacement of the soil and structure. By finding these two dependent ratios, the soil settlement can be obtained from the displacement of the structure.

Therefore, the aim is to fit a simple equation to find the ratios of the soil settlement to the total settlement $\frac{\Delta_{soil}}{\Delta_{total}}$, and displacement of structure to the total displacement $\frac{\Delta_{structure}}{\Delta_{total}}$, in order to use it to predict the settlement of soil.

In order to accommodate the behavior of the soil-structure interaction, and to reach the main objective of this research, several secondary objectives will be discussed. The stress distribution curves in the soil medium, and the volume of the soil needed to be simulated, the upper and lower limits of soil modulus of elasticity for the main two methods used by the structural and geotechnical engineers, in addition to the effect of the structural dimensions on the displacements and the displacements ratios.

1.9 Scope of work

The scope of this research is limited to square columns and footings only, for a simple structure of one column and footing, and one story two-span frame with equal spans and identical columns and footings dimensions.

Because the objective of this work is considered a guideline for the designers to find the soil settlements, many assumptions are made in order to simplify the calculations. The materials of the structure and the soil are assumed elastic, homogeneous and isotropic (Kocak, Mengi, 2000), with specific modulus of elasticity, ignoring the plastic behavior of materials. The modulus of elasticity of soil can be used as a main property and parameter for calculations, where it is considered one of the acceptable methods used to find the elastic settlement of soil (Das, 2009).

As mentioned earlier, the shallow single footing type is to be studied in this thesis. Also, the elastic settlement is the only settlement that will be discussed, ignoring the consolidating settlement effect. Because the surface displacement of soil is the one that affect the structure greatly, this research will focus on this displacement only.

1.10 Methodology

Many researches, papers and thesis were read, in order to conclude suitable literature review that represents the up to date practice in the soil-structure interaction. From which, the finite elements method was chosen to be the calculation tool to find the results for the soil-structure interaction model.

As mentioned earlier, the objective of this research is to estimate the settlements of the soil easily. Therefore, to simplify the process, the methodology that will be used to find the settlements of the soil is to link the soil settlements to the displacement of the structure as a ratio from the total displacement. This step will produce two dependent ratios, soil settlement to total displacement $\frac{\Delta_{soil}}{\Delta_{total}}$, and displacement of structure to total displacement $\frac{\Delta_{structure}}{\Delta_{total}}$.

The importance of these ratios is to be able to find the soil settlement using the displacement of the structure. By finding the displacement of the structure and by knowing the ratio of the structural displacement $\frac{\Delta_{structure}}{\Delta_{total}}$, the total displacement can be found, which from the soil settlement can be found by knowing the ratio of the soil settlement $\frac{\Delta_{soil}}{\Delta_{total}}$.

Therefore, the fitted equation will be used to find these displacement ratios. This will be done using the results from the finite element model, taking into consideration different cases of soil-structure interaction models. These results will cover a wide and practical range of parameters, which are important for typical design process.

In order to have wide range of soil types, the modulus of elasticity will be used as the reference and as the main property of the soil. By changing the modulus of elasticity of the soil, the soil settlement will change for the same external pressure by Hooke’s low. By recording the settlements of soil and the displacement of structure from every case, certain diagrams are obtained, which from equations can be fitted in order to find the displacement ratios.

1.11 Thesis outline

The research is divided into five chapters, chapter one is an introduction to the thesis, literature review, problem statement, thesis objectives and methodology. Chapter two discusses the verification of the finite elements software, to assure that the software gives accurate results, where the stresses and displacements will be compared with one of the analytical calculation methods. Also, it will discuss the soil volume that will be used in the soil-structure interaction models. Chapter three will discuss the simple structure of one column and footing, analyzing the results and fitting a general equation to calculate the displacement ratios. Chapter four is where the general equation fitted from chapter three will be tested for the frame, where the applicability of the equation and the resulting differences will be discussed. Finally, chapter five includes the conclusions from the discussed data, in addition to the equation applications and limitations. Also, recommendations for further researches are discussed at the end of the chapter.

2 Modeling and verification of soil behavior

2.1 Introduction

Estimating the increase in stress and the associated displacement caused in the soil mass due to an external loading using the theory of elasticity is an important component for the safe design of the foundations of structures (Das, 2013). Therefore, because of the importance of the soil in the system, the reactions that occurred in the soil medium duo to external forces must be studied and discussed.

Because the final fitted equations will be based on the results of the finite elements method, it is important to assure the accuracy of the results concluded from the finite elements software. Many analytical methods were developed to find the displacement and the stress at a certain point in the soil medium. Therefore, to assure the accuracy of the finite elements results, a comparison is conducted between the finite elements results and the analytical results.

For soil medium affected by vertical loads, two references are chosen for the comparison. First, the displacement at the surface of the soil will be discussed, where this location is chosen because the surface settlements are the most important displacements that affect the structure. However, because the check of the displacement is limited on the surface of the soil, it is important to assure the accuracy of the reactions of the inner elements of the soil. Therefore, the second reference is stress, where comparing the stress curves within the soil medium has the benefit of verifying the software accuracy. Also, because the materials are assumed elastic, the stress and strain are related by Hooke's law. Therefore, assuring the accuracy of the stresses gives an indication of the accuracy of the displacements.

2.2 Analytical methods

The French mathematician Boussinesq derived an equation from the theory elasticity to find the stress and displacement for area loads in three dimensional medium (Das, 2008). They have certain assumptions to be applied, where the soil is assumed to be elastic, homogenous, isotropic and weightless. Although, these assumptions are not realistic due to the soil true characteristics, which are non-homogeneous, anisotropic, in addition to the weight of the soil, which produces internal stresses. However, these assumptions are sufficient to simplify the calculations, and give acceptable results (Das, 2008). These methods give a very good indication of the displacement and stress through the soil medium, and give reasonable displacement and stress values.

To assure the accuracy of the results, the displacements at the surface of the soil are found analytically and by finite elements, and a comparison between the both results takes place in order to find out the differences and the degree of accuracy. Also, the diagrams of the stresses due to vertical loads are drawn and compared with the finite element diagrams of stresses to find the differences.

2.3 Model's assumptions

Because the materials are assumed in the analytical methods to be elastic, homogenous, isotropic and weightless, these assumptions will be adopted for the finite elements model in order to compare the results under the same conditions.

Several assumptions are developed for various conditions to find the displacement and stress for a certain point. Boussinesq assume the soil to be three-dimensional-medium, and to satisfy this assumption the soil is simulated in the software as-three dimensional-multi-nodded elements.

In nature, soil always has rigid bedrock beneath it, whatever the thickness of the flexible soil is. This assumption was adopted in the finite element, and the rigid bed rock is simulated by restraining the bottom joints with pin restraints. To eliminate the tension effect at the side of the soil elements, no restraints are assigned to the joints. However, large dimensions of soil are used, in order to simulate semi-infinite nature of soil continuity.

2.4 Mesh size selection

The most important requirement of the mesh selection is to use mesh size that gives an acceptable accuracy of the results. To test the mesh sensitivity in the results of the displacements of the multi-nodded elements, analogical comparison is conducted between two models with different mesh sizes; one with mesh size of 0.5*0.5m area and 0.5m depth, and the other with 1*1m area and 1m depth, considering the other parameters as constants. The two models are affected by area load with the same pressure value and area dimensions. The reference of the comparison is the displacement under the center of the area load for different depths.

Figure 2.1shows a comparison between the two models, where the 0.5m*0.5m area with depth of 0.5m model is named model A, and the 1m*1m area with 1m depth is named model B. From the figure, it is obvious that the differences between the two models are insignificant, where the slope equals 1 and the coefficient of determination (R^2) approximately equals 1 too.

Depending on these results, the mesh system used for the models in this research has elements with different sizes. The mesh size decreases when approaches the load source to have more accurate results, while the mesh size increases by moving away from the load source, in order to assure an acceptable accuracy, and decrease the analysis duration.

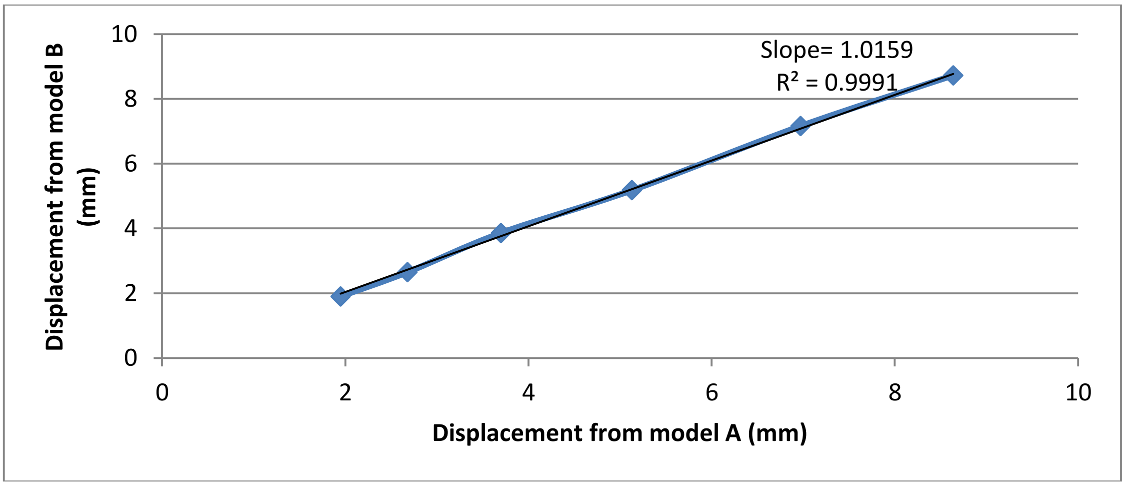

Figure 2.1: The displacement from model A versus the displacement from model B, as an indication of the mesh size effect on the displacement of the multi-nodded elements.

2.5 Results and discussions

This section will show the results of many models, and discusses the comparison between the analytical methods and the finite elements method.

2.5.1 Soil displacement

Using the theory of elasticity principles, the researchers have derived an equation to find the elastic settlement of soil at any depth of the soil. Equation 2.1 is used to find the settlement of soil (Das, 2008).

$$\Delta = \frac{qB}{2E}(1-\upsilon^2)\left(I_9 - \left(\frac{1-2\upsilon}{1-\upsilon}\right)I_{10}\right) \tag{2.1}$$

Where:

q: external pressure value.

Δ: elastic settlement of soil.

B: width of area load.

E: modulus of elasticity of soil.

υ: Poisson's ratio.

I_9: influence factor that depends on the dimensions of the area load (Das, 2008).

I_{10}: influence factor that depends on the dimensions of the area load and the depth of the targeted point (Das, 2008).

However, because the focus of this research is on the surface settlement of soil, the depth of the reading point is zero. Therefore, the influence factor I_{10} has zero value, and the equations used to find the displacement at the surface of the soil for the corner and the center of the area load are presented in Equation 2.2 and Equation 2.3 respectively (Das, 2008).

$$\Delta_{corner}=\frac{qB}{2E}(1-\upsilon^2)*I_9 \tag{2.2}$$

$$\Delta_{center}=\frac{qB}{E}(1-\upsilon^2)*I_9 \tag{2.3}$$

Poisson's ratio is taken to be 0.3, which represents the average of the ratios of the soil, and this ratio exists in all the soil types. Using Equation 2.2 and Equation 2.3, it is concluded that the maximum error for the higher and lower ratio does not exceed 15%, which is acceptable.

Many models were assumed with different dimension variables and different soil modulus of elasticity, in order to generate results for the comparison between the finite elements results and the analytical results. For these models, the influence factors are found, and using Equation 2.2 and Equation 2.3the settlements of soil are calculated. These models are simulated in the finite elements program, and the results are obtained.

Figure 2.2 shows a comparison between the analytical method and the finite elements method, where the analytical results are presented versus the finite elements results, where by noticing the slope of the curve, the accuracy of the finite elements results can be obtained. From Figure 2.2, the slope of the trend line has a value of 1.15 which is higher than the value 1. This means there is approximately 15% error, while the coefficient of determination R^2 equals 0.976. This is an indication that the finite elements results have acceptable accuracy.

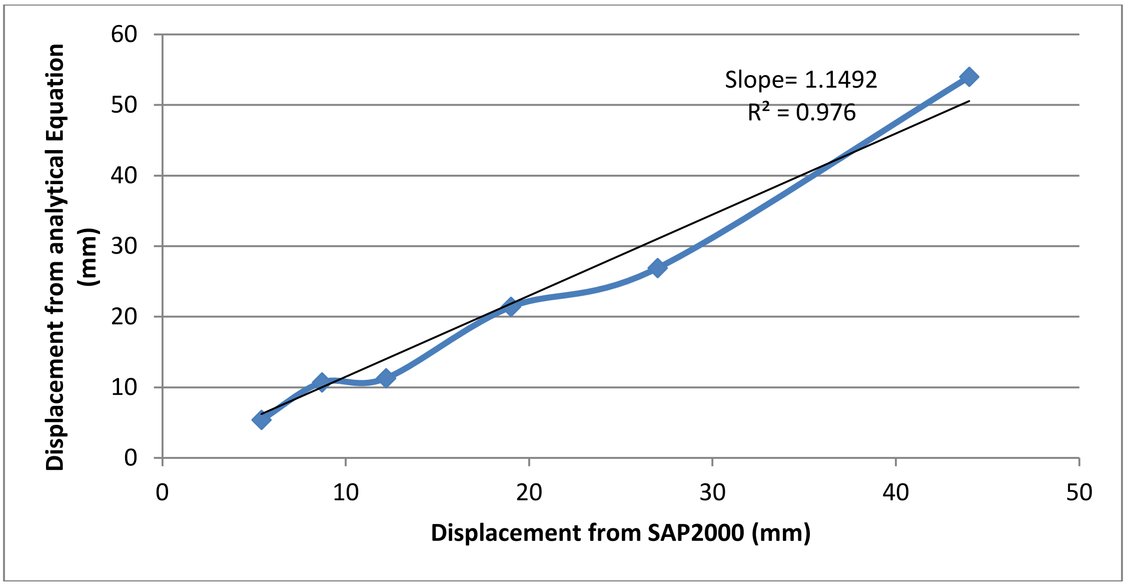

Figure 2.2: Displacement of soil from SAP2000 versus the displacement from the analytical equation 2.2 and 2.3.

2.5.2 Soil stresses

Despite the conclusion from the previous section that confirms the accuracy of the displacement of soil at the surface level, the stresses within the soil medium must be checked, in order to assure the accuracy of the finite elements stress distribution.

An equation was developed by Boussinesq to find the stresses due to vertical area loads for a specific reading point in three dimensional medium soils (Das, 2008). This method is very important because it shows the stress curves for three dimensional elements just like single footings and matt foundations. Equation 2.4 was derived by Boussinesq to find the stress at a point due to the area load (Das, 2008).

$$\sigma_z = q * I_7 \tag{2.4}$$

Where:

q: load per unit area.

I_7: influence factor, which can be expressed by Equation 2.5.

$$I_7 = \frac{1}{4\pi}\left[\frac{2mn\,(m^2+n^2+1)^{1/2}}{m^2+n^2+m^2n^2+1} * \frac{m^2+n^2+2}{m^2+n^2-1} + \tan^{-1}\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2-m^2n^2+1}\right] \quad (2.5)$$

Where:

$m = \frac{B}{z}$

$n = \frac{L}{z}$

B: width of the area load from the edge to the reading point.

L: length of the area load from the edge to the reading point.

z: depth of the reading point.

2.5.1.1 One area load

An example is used to find the results, where a square area with 2B width dimension is used, with external pressure value q, and a depth in term of the constant B. Figure 2.3 shows the stress distribution curve for the calculated results using the analytical method, and Figure 2.4 shows the distribution curve due to the finite element analysis using the program SAP2000, noticing that this figure represents a cross section at the center of the area load and the stress is a ratio from the total external pressure q; i.e. stress ratio $= \frac{\text{stress from the finite elements model at a certain point}}{\text{external pressure affects the soil}}$.

Figure 2.5 shows a diagram that compares the two results, where $\left(\frac{Finite\ elemnt\ results}{Analytical\ results}\right)$ is plotted versus the depth and the width of the soil. Which from, it is obvious that the difference ratio has values between 0.85 and 1.1, with approximate error of 15%, which is considered acceptable.

Figure 2.6 and Figure 2.7 show the stress ratios from the finite elements and the analytical calculations versus the depth directly under the center and the edge of the area load respectively, which from it is concluded that small errors are occurred and the results are acceptable.

However, significant errors are noticed near the edges of the area load. This occurred because this finite elements software considers the three dimensional multi-nodded elements as total elastic element, which gives the same results in tension and compression. However, this behavior does not imply on the soil, which have approximately no tension capacity, especially the loose soil. Therefore, the soil volumes near the area load will be affected by tension forces because of the area load. Despite that, the effect of these stresses on the main stress curves is negligible, and can be ignored safely as they have insignificant effect on the main stress flow.

2.5.1.2 Two area loads

Another example of two area loads with the same dimensions and assumptions as the previous example is used to find the effect of the nearby area, with a distance B between the two area loads. Figure 2.8 shows the stress diagram of the calculated results using the analytical method, and Figure 2.9 shows the diagram due to the finite element analysis using the program SAP2000. Figure 2.10 shows a plot of $\left(\frac{Finite\ elemnt\ results}{Analytical\ results}\right)$ versus the depth and width of the soil medium, where the comparison between the two results gives $\left(\frac{Finite\ elemnt\ results}{Analytical\ results}\right)$ results between 0.85 and 1.15, giving a percentage of error of 15%.

To assure the results, Figure 2.11 and shows the stress distribution for the change of the depth directly under the center of one of the area loads and Figure 2.12 shows the stress distribution for the change of the depth at the far center of one of the area loads edge, which gives acceptable accuracy with small errors.

On the other hand, a certain volume between the area loads gives significant errors, and the finite elements behavior did not match the analytical behavior. As mentioned in the previous section, the volumes near the area loads are affected by tension forces due to the area loads, which will affect the stresses values, giving odd values. Moreover, because the soil volume between the two area loads is affected by the both areas, the effect is doubled, which magnify the difference between the finite elements results and the analytical results. However, these volumes can be neglected, because their effect on the main stress curves is negligible. Thus, these volumes will be cut later in the frames-soil interaction chapter.

2.6 Soil volume selection

From the previous section, it is obvious that the stress is dissipating when moving away from the pressure source. Using this observation, the needed volume of the soil for the finite elements model can be obtained. From Figure 2.3, Figure 2.4, Figure 2.8 and Figure 2.9 it can be noticed that for the depth 5B and the soil width of 2B near the area load, the stress percentage reaches approximately 10%. Assuming this stress ratio as the ignorable threshold, it is concluded that the volume of soil needed for the finite elements model must be higher than (4B*4B area with 5B depth).

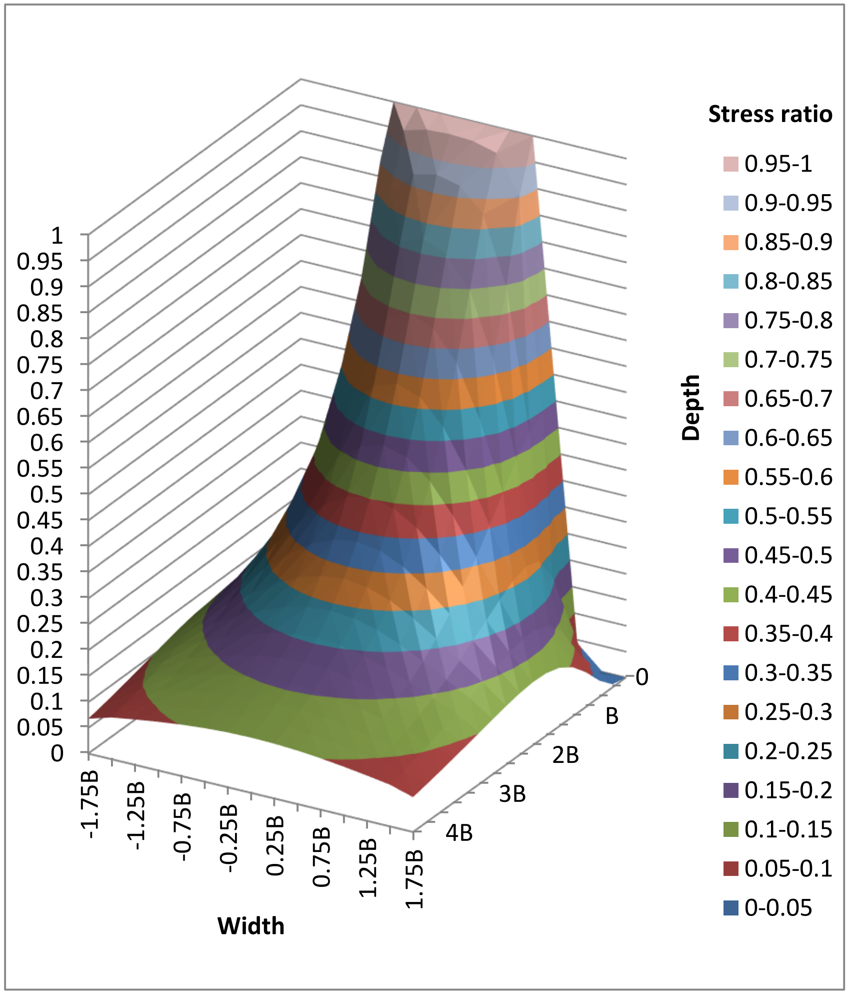

Figure 2.4: Stress curves for an area load due to analytical calculations.

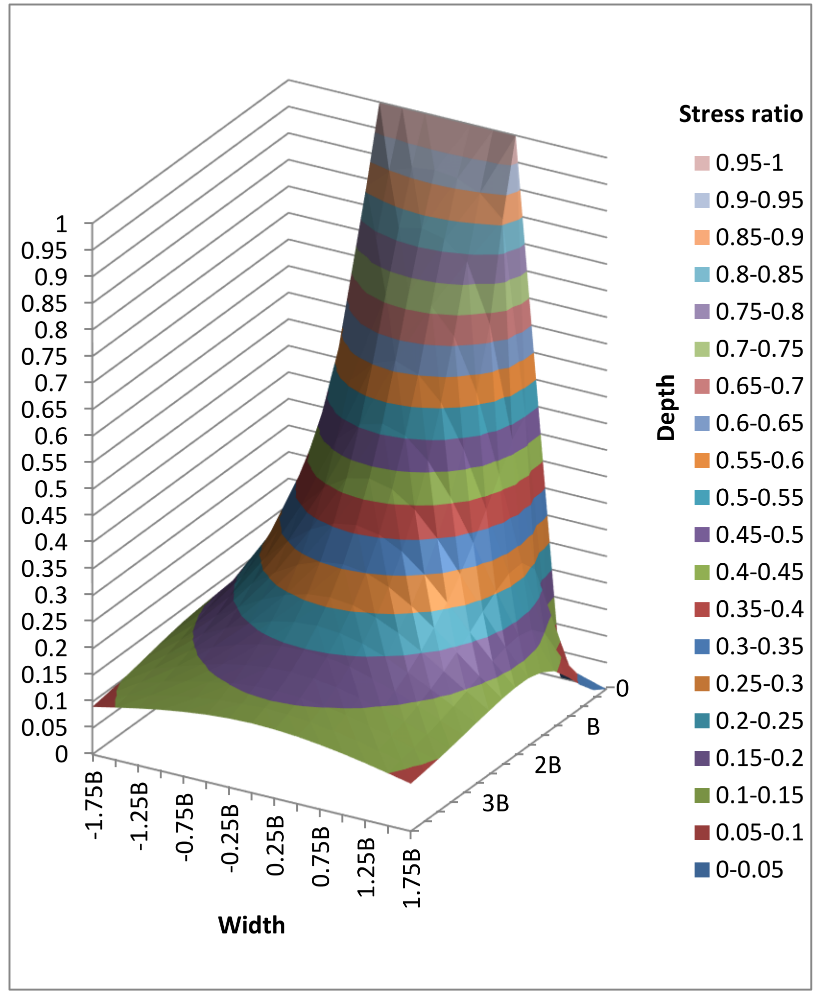

Figure 2.3: Stress curves for an area load due to finite element analysis using SAP2000.

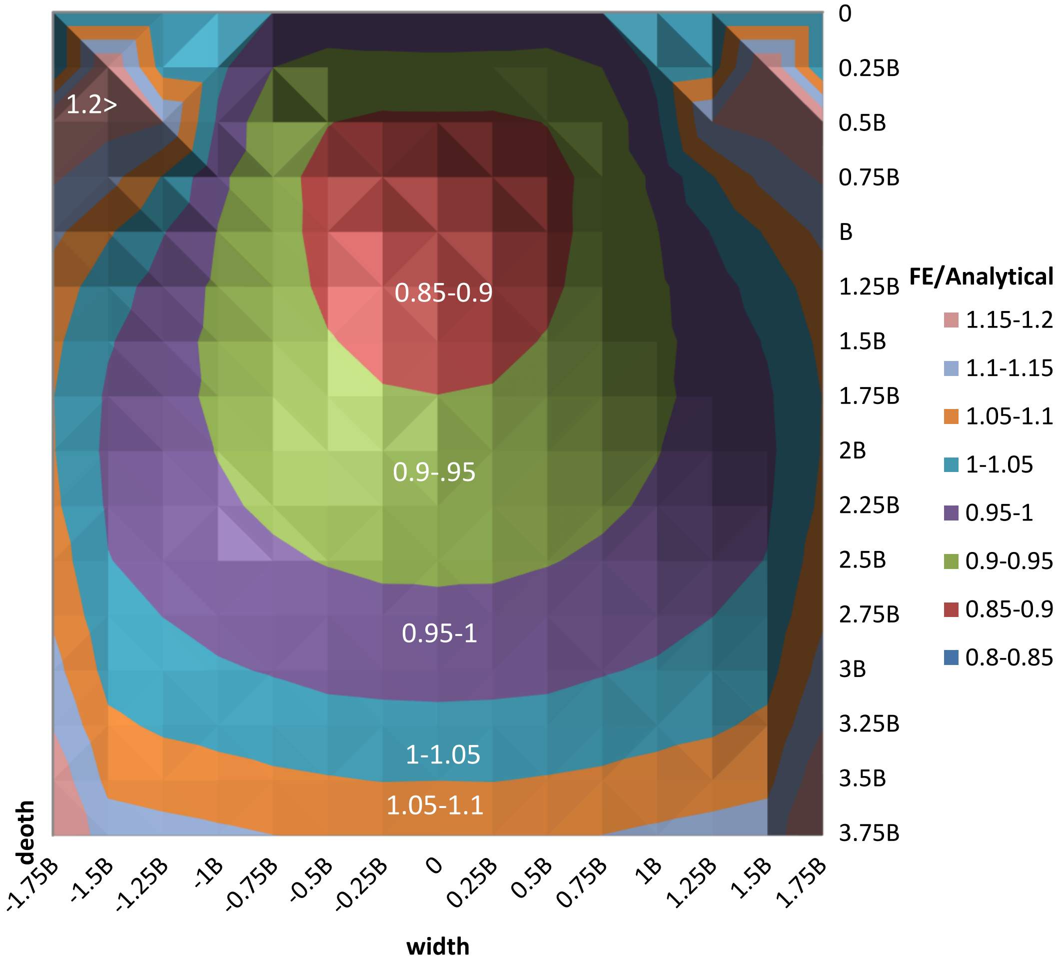

Figure 2.5: Comparison between the analytical and the finite element methods as a ratio to the analytical results for an area load.

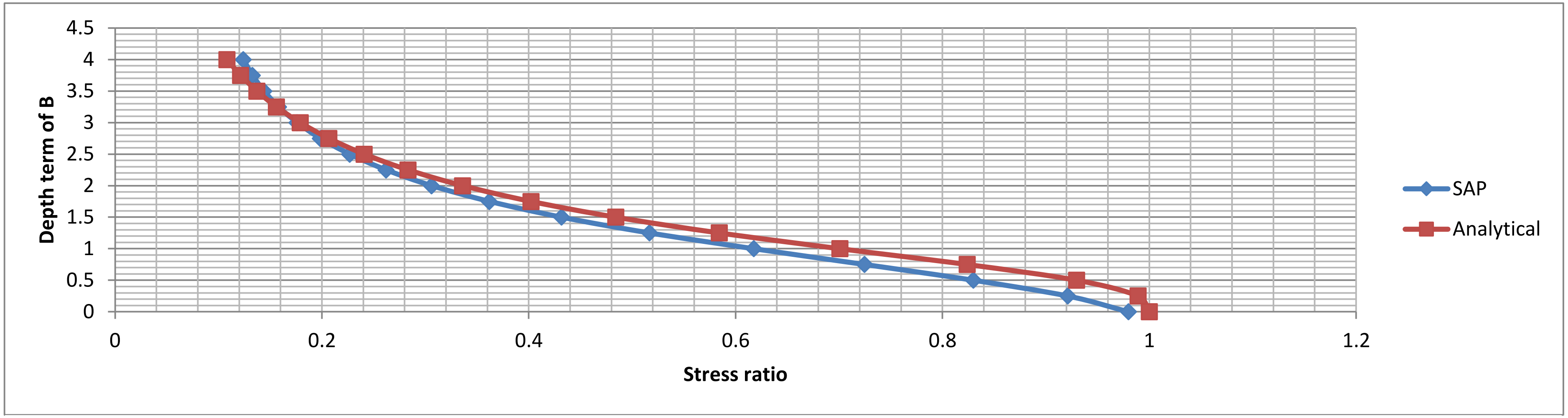

Figure 2.7: Comparison between the analytical stress and the finite element stress under the center of the area load.

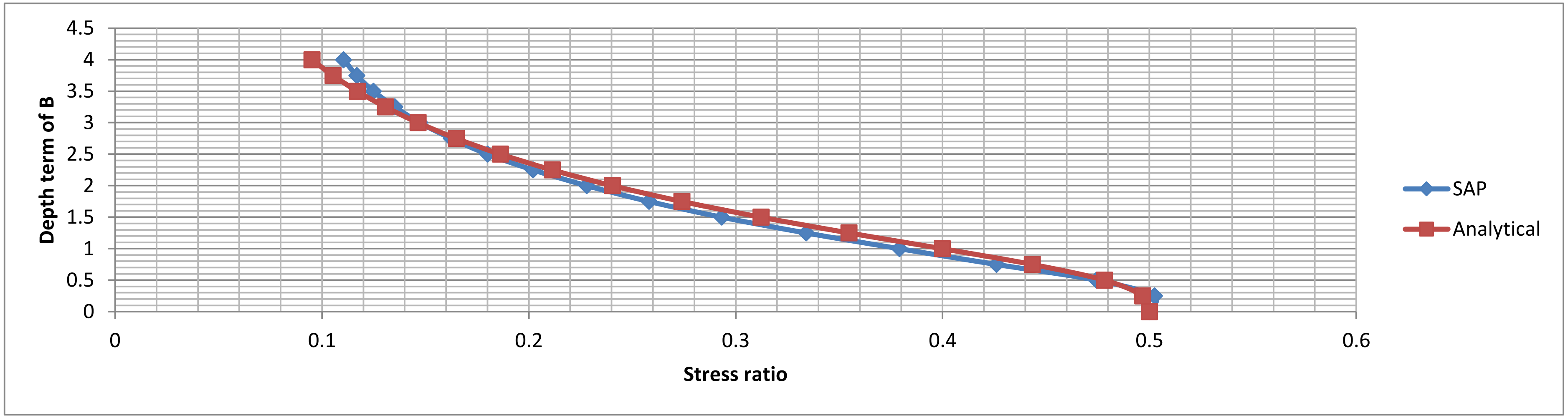

Figure 2.6: Comparison between the analytical stress and the finite element stress under the corner of the area load.

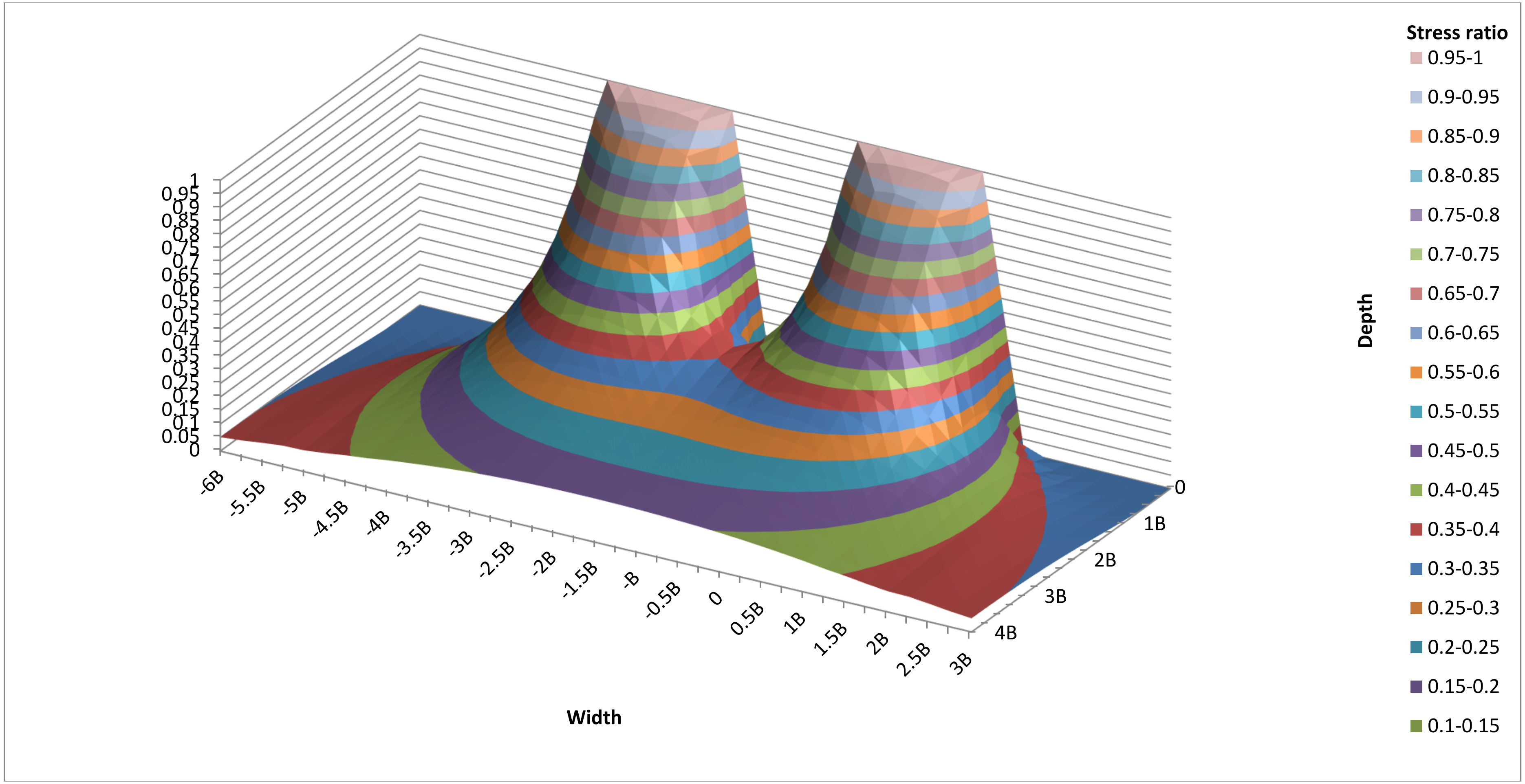

Figure 2.8: Stress diagram for two area loads due to analytical calculations.

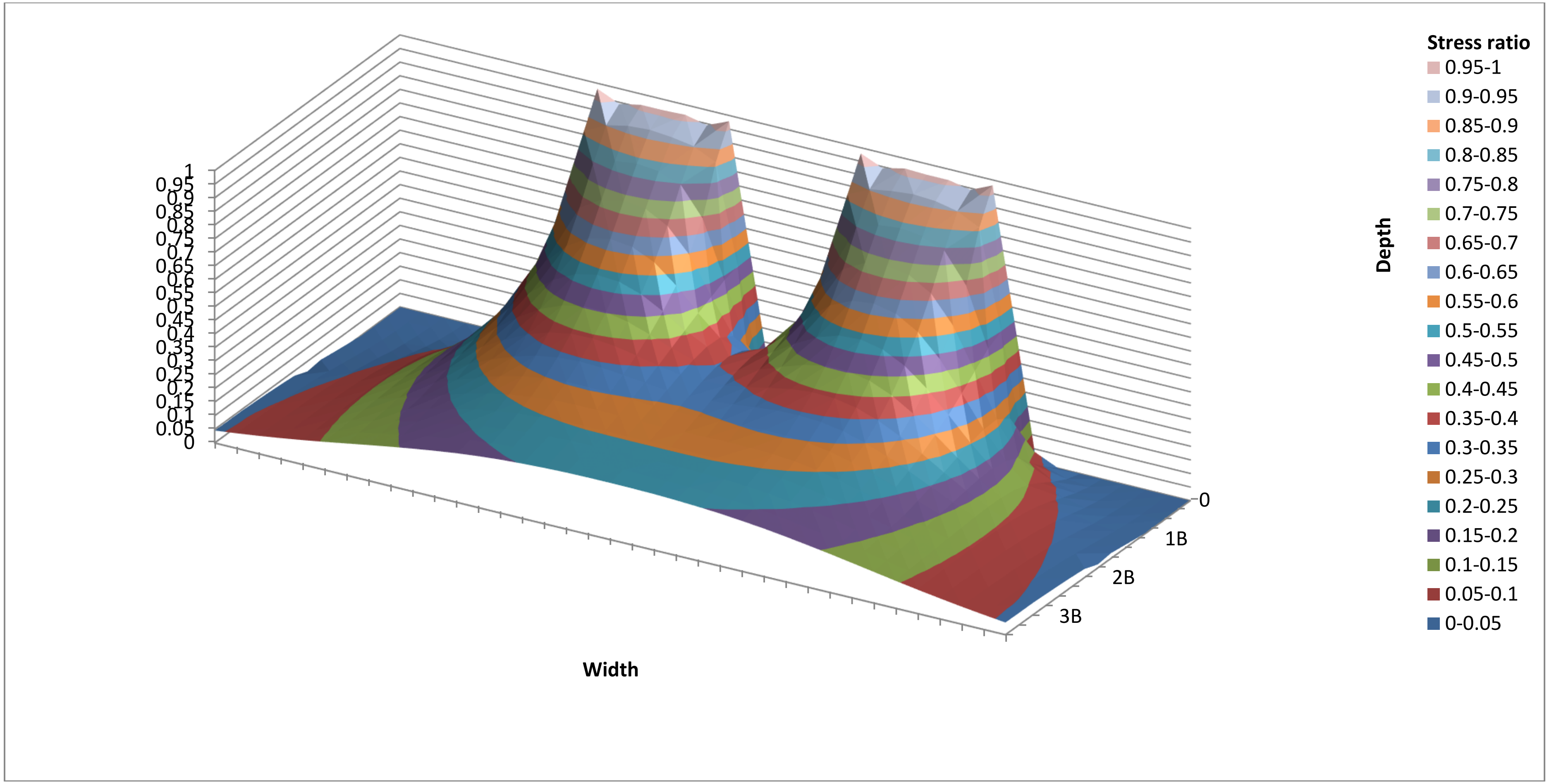

Figure 2.9: Stress diagram for two area loads due to finite element calculations

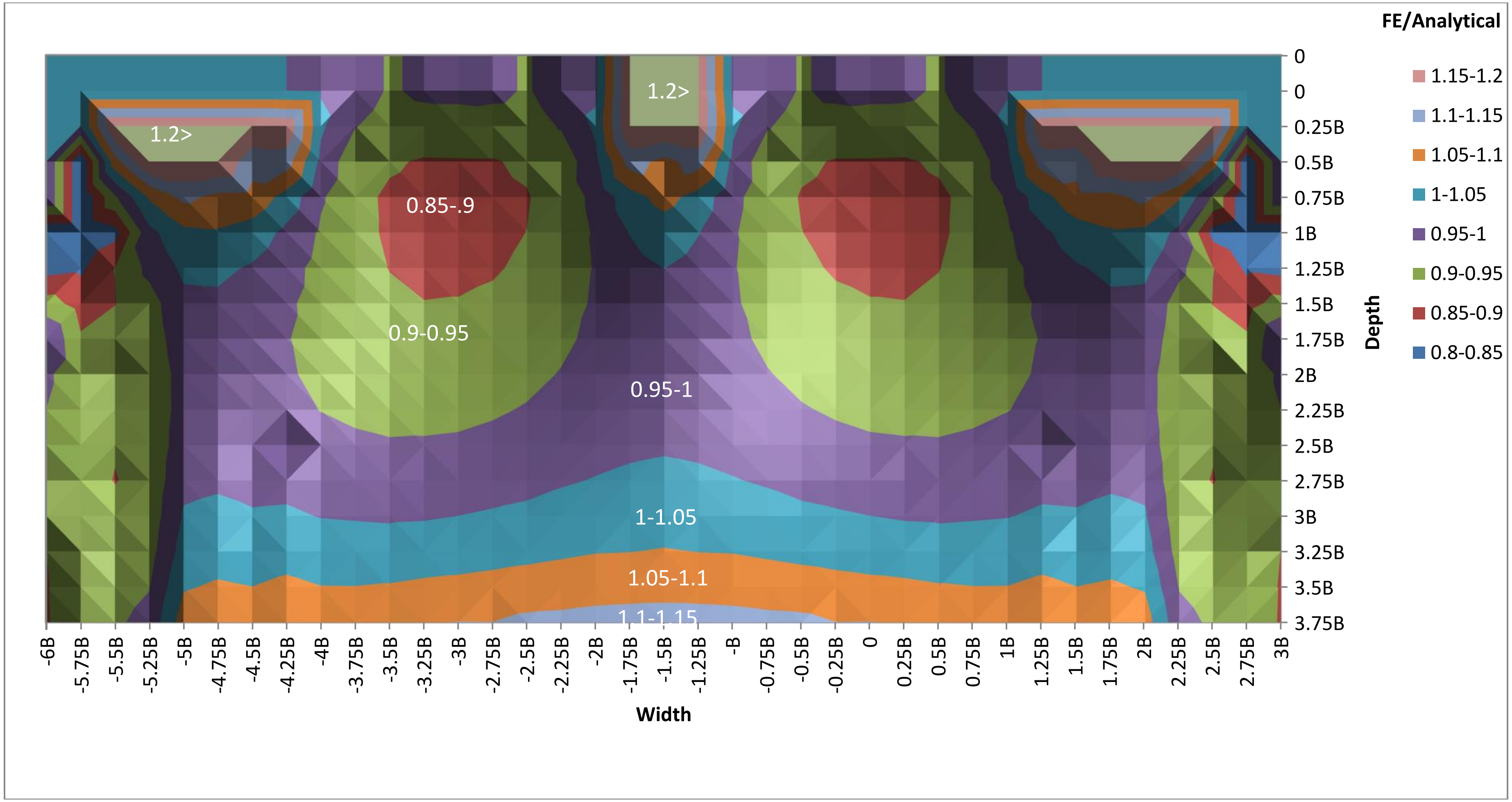

Figure 2.10: Comparison between the analytical and the finite element methods as a ratio to the analytical results for two area loads.

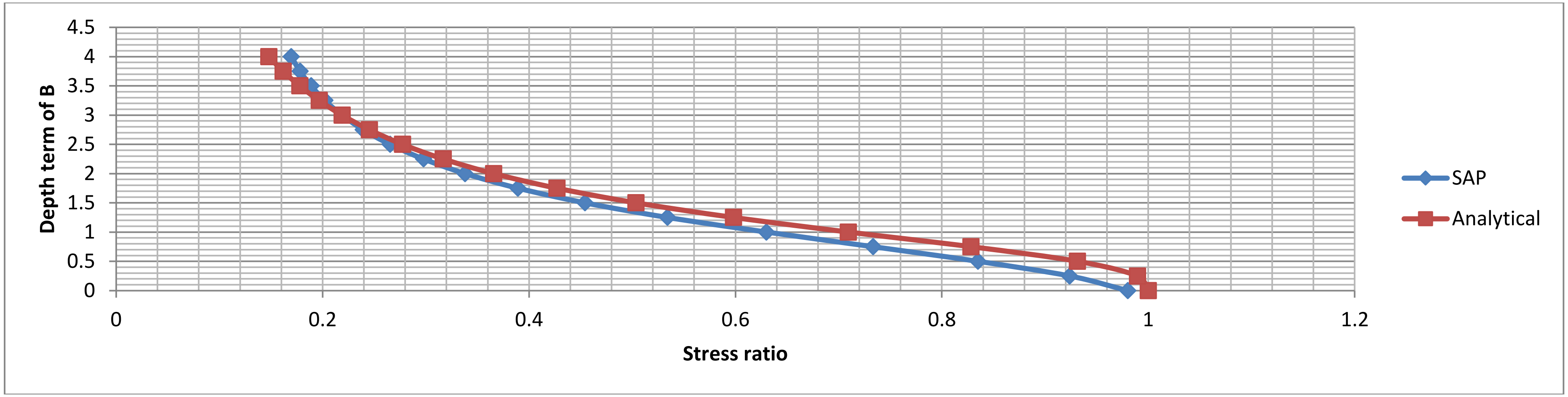

Figure 2.11: Comparison between the analytical stress and the finite element stress under the center of the right area load.

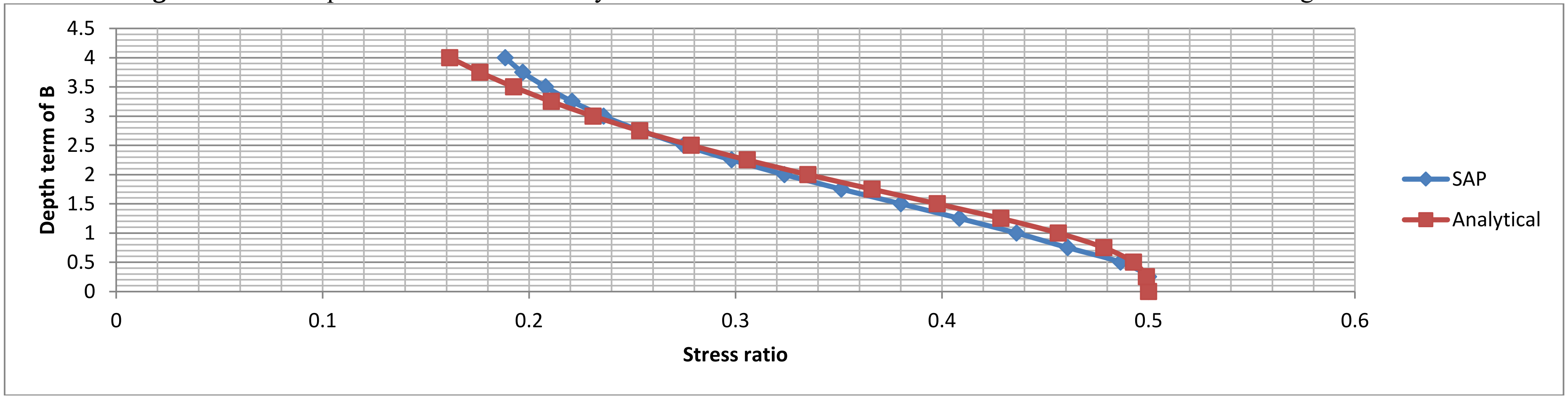

Figure 2.12: Comparison between the analytical stress and the finite element stress under the left corner of the right area load.

3 Soil-structure displacement ratios for simple model of column and footing due to vertical loads

3.1 Introduction

To obtain the results of the soil-structure displacement ratios, the finite elements method is used, where the commercial program SAP2000 will be the tool to do the analysis.

After finding the results and calculating the displacement ratios, simple equations will be fitted, to be used as simplified guidance for practical and conceptual design phases.

3.2 Structural model

The adopted model is a simple model of a square column with vertical stress assigned to the top of the column, and a square single footing placed on the soil. The square shape is used to simplify the calculations and to reduce the number of variables. This model is chosen because of its simplicity and because the parameters are manageable. Both the structure and soil are defined as three-dimensional multi-nodded elements in the finite elements program SAP2000. Figure 3.1 shows a sketch for a representative model and Figure 3.2 shows a representative meshed model.

The main parameters that affect the model are shown in Figure 3.1 and are clarified in the following points:

- The dimension of footing side, *l*.
- The dimension of column side, *c*.

- Depth of footing, *d*.
- Height of column, *h*.
- Stress assigned to the column, *σ*.
- Dimensions of soil volume, which are assumed as 25m*25m area with 15m depth.

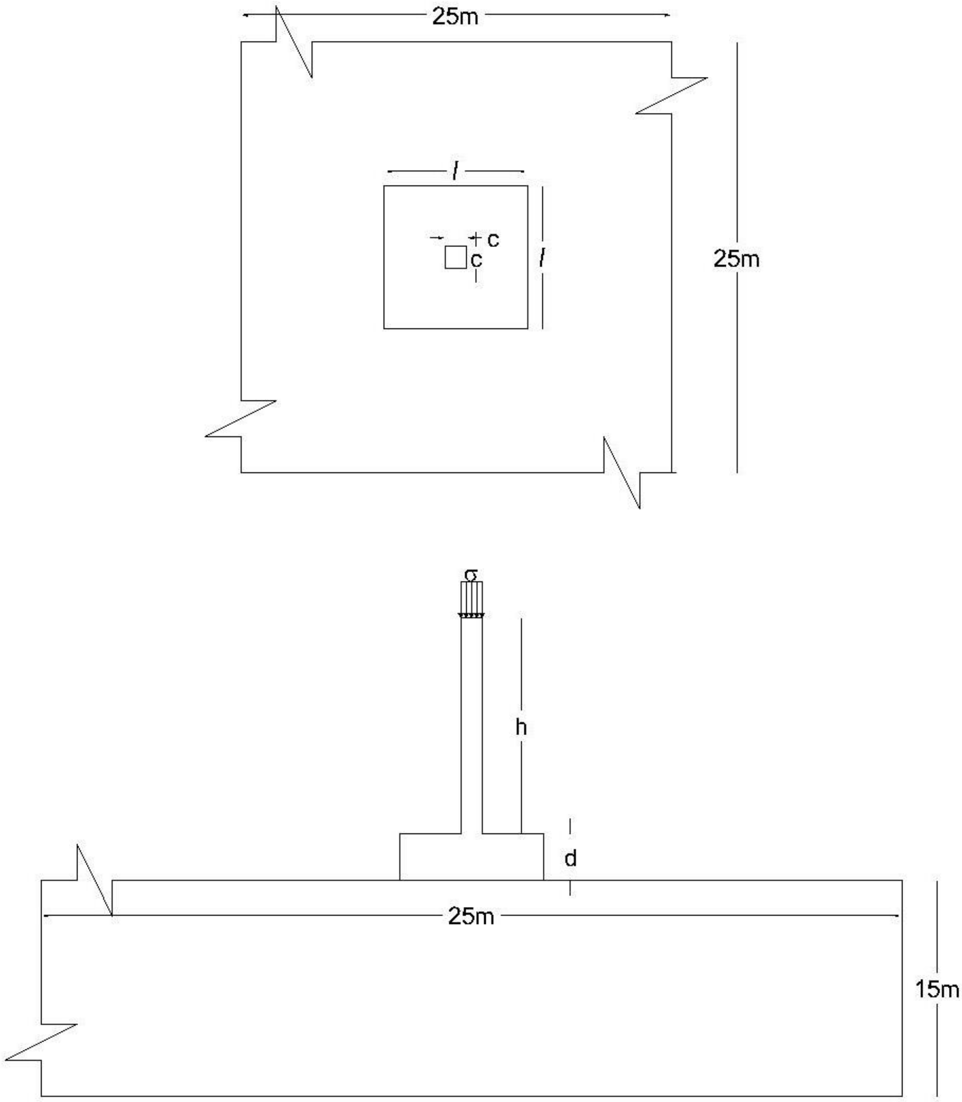

Figure 3.1: Sketch shows the model's parameters.

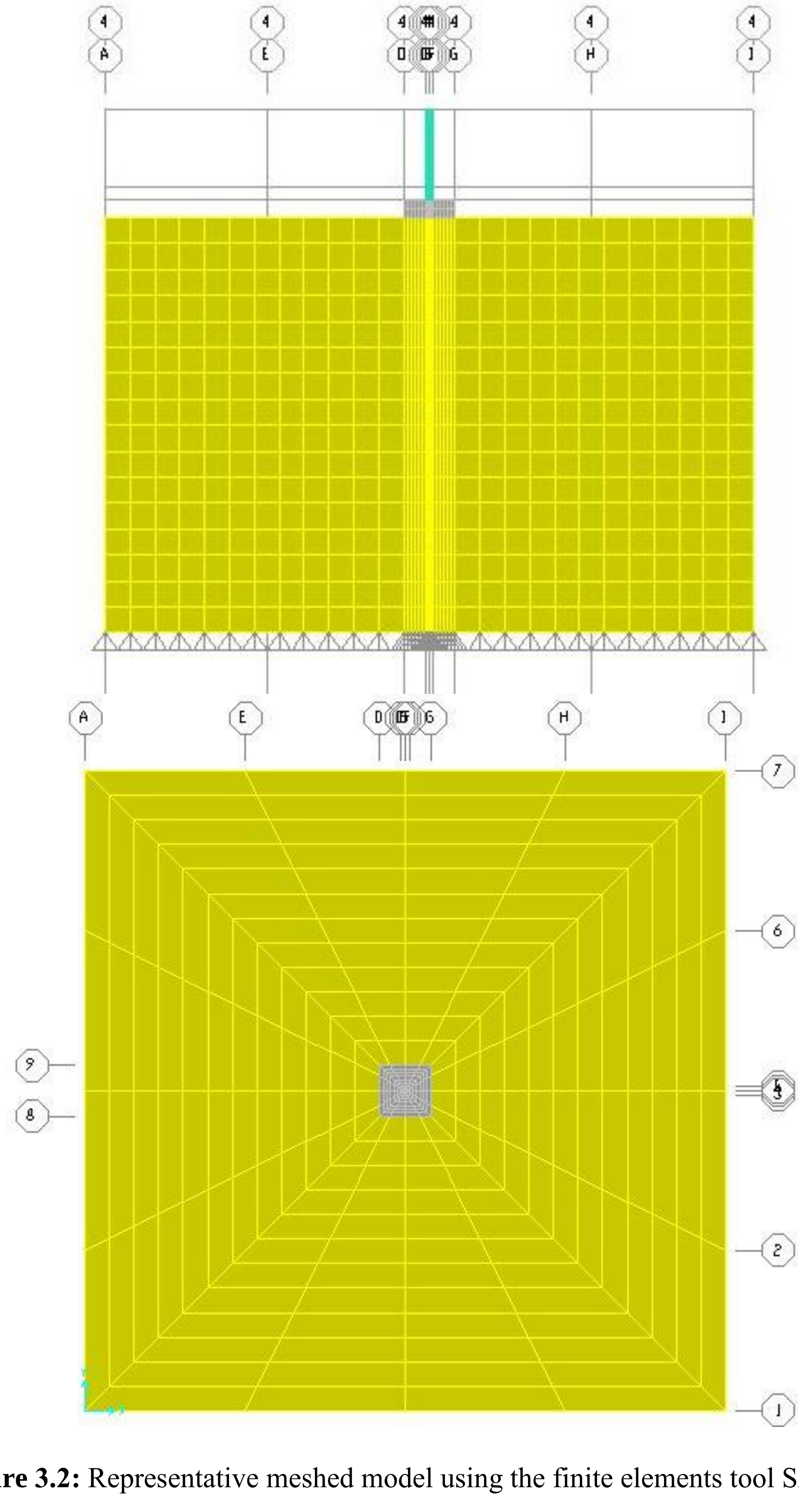

Figure 3.2: Representative meshed model using the finite elements tool SAP2000.

The soil dimensions are chosen with a value higher than the minimum dimensions for the minimal stress distribution. As mentioned in Chapter 2, the stress distribution for the area loads was discussed, and by assuming the ignorable stress threshold to be 10% of the total stress, and based on Figure 2.6 and Figure 2.7, it was noticed that the minimum soil depth is approximately 5B, where $B = \frac{l}{2}$ and the minimum dimension of soil near the footing is 2B, noticing that the footing side dimension is 2m. Thus, the proposed soil volume is more than the minimum volume.

According to the (CSI, 2010) manual, the solid element is an eight node element for modeling three dimensional structures and solids, which is based upon an isoperimetric formulation that includes nine optional incompatible bending modes. Each element has its own coordinate system for defining material properties and loads and for interpreting output.

The size of the mesh is selected from previous experiences based on achieving sufficient accuracy and to reduce the duration of analysis, as was clarified in Section 2.4. Trial and error approach is followed by changing mesh size until it is conceded that stress and strain results do not vary significantly. The mesh sizes are selected to gradually decrease when moving towards the structure in order to satisfy acceptable accuracy of the results.

3.3 Material model

The materials of soil and structure are assumed fully elastic, homogeneous and isotropic in order to simplify the model (Kocak and Mengi, 2000). The soil is assumed to be dry with no water pores in order to find the immediate settlement only and ignore the consolidation settlement (Bowles, 1982). According to (Bowles, 1982) this method is “used for all fine-grained soils including silts and clays with a degree of saturation S $\leq$ 90 percent and for all coarse-grained soils with a large coefficient of permeability”. Also, (Holtz and Kovaks ,1981) stated “the immediate, or distortion, settlement, although not actually elastic is usually estimated by using elastic theory”. The materials used for soil vary from a very soft soil of 5MPa modulus of elasticity, to a very stiff soil with modulus of elasticity of 10000MPa.

The structural material for footings and columns is assumed to be concrete, with modulus of elasticity of 24500 MPa, which is referred to as $E_{structure}$. The materials densities are assumed to be zero for the soil and the structure, to assure the rest condition, in order to have zero displacements before the stresses applied. Because the model is assumed elastic, this will have no effects on the results, as the target of this research is the relative values.

3.4 Basic assumptions

In order to normalize the results, some variables are normalized as follows: the ratios of length of footing side to depth of footing $\left(\frac{l}{d}\right)$, length of column side to length of footing side $\left(\frac{c}{l}\right)$ and the ratio of the soil modulus of elasticity to the concrete modulus of elasticity $\frac{E_{soil}}{E_{structure}}$. The other parameters were assumed constants.

The materials are assumed elastic, homogeneous and isotropic to simplify the analysis, and because this assumption is enough to satisfy the purpose of this research.

Large soil dimensions are assumed, with 25m*25m area and15m depth, to neglect the effect of the artificial boundaries. No side restrains were assigned; because the amount of stresses at the edges is negligible. Beneath the depth of the soil, a layer of rigid bedrock is assumed. To accomplish that, the base joints were restrained with pin restraints.

The interface between footing and soil is assumed continuous, and separation between joints of footing and soil because of deformations due to shear stresses was ignored. This was assumed because the frictional effects on shear are very small and negligible.

The reinforcing steel is ignored as the calculations are usually based on the gross section of the concrete.

3.5 Procedure

As mentioned in Chapter 1, the soil settlement is to be found using specific ratios: the displacement of structure to total displacement $\frac{\Delta_{Structure}}{\Delta_{total}}$ and the ratio of the displacement of soil to the total displacement $\frac{\Delta_{soil}}{\Delta_{total}}$. In order to find the displacement ratios for a certain model, the displacement values must be found.

By analyzing the models for each set of parameters, and by finding the total displacement Δ_1 and the soil displacement Δ_2 as shown in Figure 3.3, the displacement of structure can be found by subtracting the two values, $\Delta_{st}=$

$\Delta_1 - \Delta_2$. Then the displacement ratios can be found. Δ_2 is taken at the soil surface level at the center of the footing to obtain the maximum settlement of soil.

To conduct parametric study, the same model is analyzed using different dimension parameters for different soil parameters, where for a certain $\left(\frac{l}{d}\right)$ value many $\left(\frac{c}{l}\right)$ values were used, and for a certain $\left(\frac{c}{l}\right)$ value all the proposed soil materials that were mentioned in Section 3.3 were used. Table 3.1 shows the run cases for models simulated in SAP2000.

The previous procedure is repeated for $\left(\frac{l}{d}\right)$ values of 3, 6 and 8, and for $\left(\frac{c}{l}\right)$ values of 0.15, 0.2, 0.25 and 0.3. The column's height is assumed constant with the value of 3 meters, and the stress is assumed constant, with the value of 6MPa, where this value represent the average service load affecting the columns.

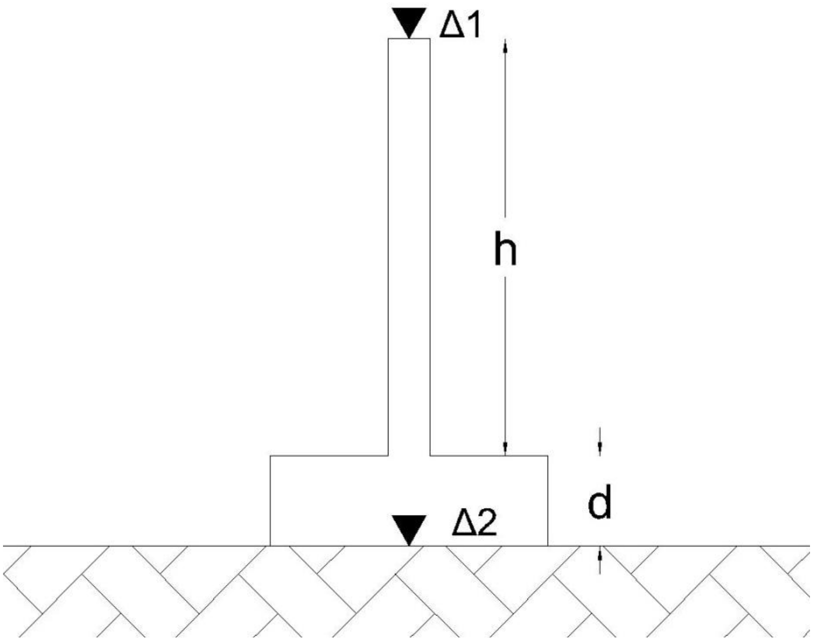

Figure 3.3: Total displacement (Δ_1), and soil displacements (Δ_2).

After finding the displacement of soil, displacement of structure and the total displacement from all the models, the ratios of soil and structure displacement to the total displacement are calculated. Afterwards, the resulting curves for each set of parameters are drawn to study the relation between the ratio of stiffness, which is at the horizontal axis, and the displacement ratio, which is the vertical axis. Two types of curves were drawn: the displacement of soil ratio and the displacement of structure ratio. The diagrams will help explaining the relationship between the two curves.

Table 3.1: Run cases

Case Number	(l/d)	(c/l)	E soil	Case Number	(l/d)	(c/l)	E soil
1	3	0.15	5	33	6	0.15	5
2			10	34			10
3			50	35			50
4			100	36			100
5			500	37			500
6			1000	38			1000
7			5000	39			5000
8			10000	40			10000
9		0.2	5	41		0.2	5
10			10	42			10
11			50	43			50
12			100	44			100
13			500	45			500
14			1000	46			1000
15			5000	47			5000
16			10000	48			10000
17		0.25	5	49		0.25	5
18			10	50			10
19			50	51			50
20			100	52			100
21			500	53			500
22			1000	54			1000
23			5000	55			5000
24			10000	56			10000
25		0.3	5	57		0.3	5
26			10	58			10
27			50	59			50
28			100	60			100
29			500	61			500
30			1000	62			1000
31			5000	63			5000
32			10000	64			10000

Table 3.1: Run cases (cont)

Case Number	(l/d)	(c/l)	E soil
65	8	0.15	5
66			10
67			50
68			100
69			500
70			1000
71			5000
72			10000
73		0.2	5
74			10
75			50
76			100
77			500
78			1000
79			5000
80			10000
81		0.25	5
82			10
83			50
84			100
85			500
86			1000
87			5000
88			10000
89		0.3	5
90			10
91			50
92			100
93			500
94			1000
95			5000
96			10000

3.6 Results and discussions

After analyzing the models, the displacement of structure, displacement of soil and the total displacement are found and tabulated in order to calculate the displacement ratios. Table 3.2 shows a representative sample of the results and calculations for a model with $\left(\frac{l}{d}\right)$ value 6 and $\left(\frac{c}{l}\right)$ value 0.15, where two types of displacement ratios were found, as will be explained in the following sections.

3.6.1 General behavior of displacement ratios study

After doing the simulations, curves are drawn representing the relationship between the ratio of the displacement of structure to total displacement $\frac{\Delta_{Structure}}{\Delta_{total}}$ and the ratio of the displacement of soil to the total displacement $\frac{\Delta_{soil}}{\Delta_{total}}$ versus the modulus of elasticity ratio $\frac{E_{soil}}{E_{structure}}$. Figure 3.4 shows $\frac{\Delta_{soil}}{\Delta_{total}}$ and $\frac{\Delta_{Structure}}{\Delta_{total}}$ curves for the model with values $\left(\frac{l}{d}\right) = 6$ and $\left(\frac{c}{l}\right) = 0.15$. It can be noticed that $\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}}$ curve starts with approximate value 1 for $\frac{E_{soil}}{E_{structure}}$ value of $2 * 10^{-4}$ and decreases to a value of approximately zero for $\frac{E_{soil}}{E_{structure}} = 2$. While $\frac{\Delta_{\text{Structure}}}{\Delta_{\text{total}}}$ curve is the mirror of the previous observation, where it starts with zero value at modulus of elasticity ratio of $2 * 10^{-4}$ and increases to approximately 1 for $\frac{E_{soil}}{E_{structure}} = 2$.

As was stated earlier, two main assumptions are used by structural and geotechnical engineers, the rigid-structure flexible-soil assumption, which is used by the geotechnical engineers, and the flexible-structure rigid-soil assumption, which is adopted by the structural engineers.

Figure 3.4 can help to explain these assumptions and clarify the boundaries for each assumption. The rigid structure assumption is achieved when the displacement of the structure is very small and negligible when compared to the soil displacements. This means that the soil displacement is the only significant aspect. Therefore, $\frac{\Delta_{soil}}{\Delta_{total}}$ value must equal approximately 1. This occurs when the soil is very soft, which gives very small value of $\frac{E_{soil}}{E_{structure}}$, and from Figure 3.4 it is obvious that $\frac{\Delta_{soil}}{\Delta_{total}}$ for the small value of $\frac{E_{soil}}{E_{structure}}$ approximately equals 1, and because $\Delta_{\text{structure}}$ values are very small and approximately zero, $\frac{\Delta_{Structure}}{\Delta_{total}}$ value goes to zero. In other words, the assumption can be used safely for soil modulus of elasticity ratio of $4 * 10^{-4}$ or less, which includes soil classification of very soft and soft clay, silt clay, silt sand and silt soils (Geotechdata, 2016).

On the other hand, the rigid soil flexible structure assumption can be examined by observing the behavior of $\frac{\Delta_{Structure}}{\Delta_{total}}$ curve. This assumption can be applied when the displacement of soil is very small and negligible when compared with the displacement of structure, which gives 1 as value for $\frac{\Delta_{Structure}}{\Delta_{total}}$. To have a very small displacement values for soil, the soil must have high modulus of elasticity, which gives high $\frac{E_{soil}}{E_{structure}}$ value. As seen in Figure 3.4, for high $\frac{E_{soil}}{E_{structure}}$ values the $\frac{\Delta_{Structure}}{\Delta_{total}}$ value is approximately 1.

Table 3.2 Represented sample shows the results and calculations of SAP2000 model with $\left(\frac{l}{d}\right) = 6$ and $\left(\frac{c}{l}\right) = 0.15$

E Soil MPa	Δ total mm	Δ Soil mm	Δ Structure mm	E Soil/ E Structure	Δ Soil/ Δ Total	Δ Structure/ Δ total
5	33.35	32.40	0.93	2.0E-04	0.97	0.03
10	17.30	16.35	0.93	4.0E-04	0.95	0.05
50	4.44	3.51	0.93	2.0E-03	0.79	0.21
100	2.81	1.88	0.93	4.0E-03	0.67	0.33
500	1.45	0.50	0.93	2.0E-02	0.35	0.65
1000	1.23	0.30	0.93	4.0E-02	0.25	0.75
5000	1.02	0.09	0.93	2.0E-01	0.09	0.91
10000	0.99	0.06	0.93	4.0E-01	0.06	0.94

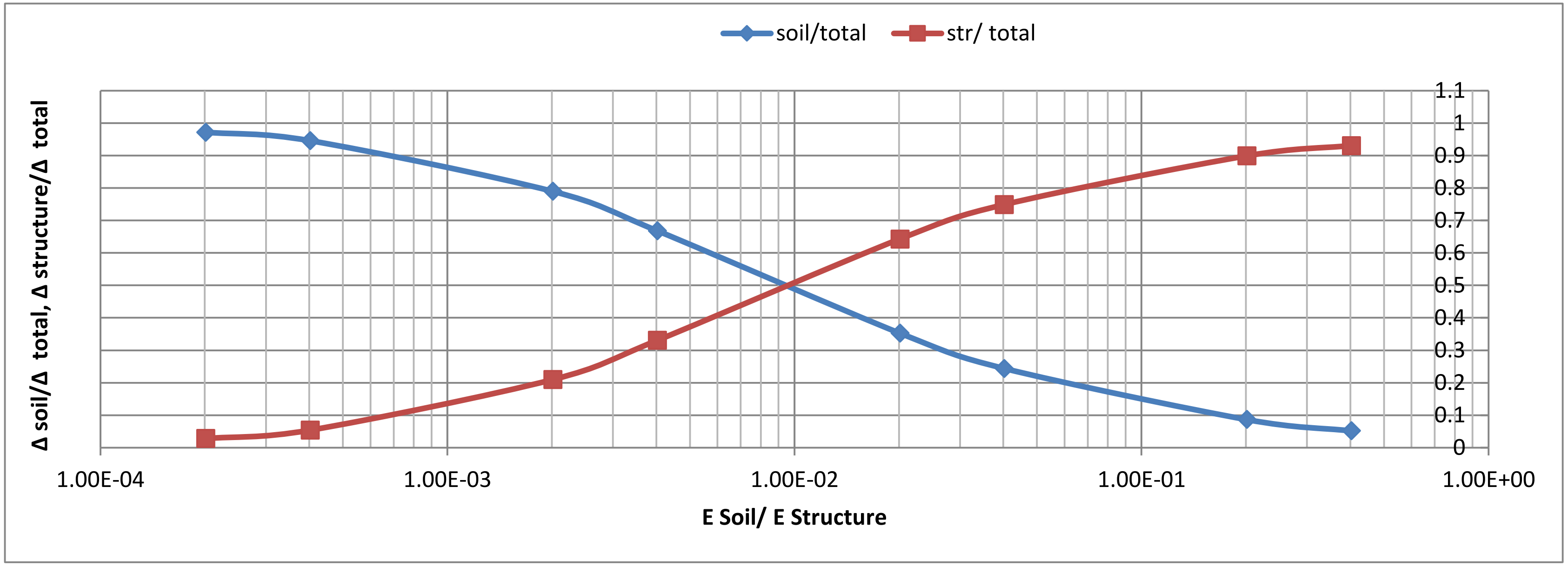

Figure 3.4 : A representative diagram shows Δsoil/Δtotal and Δstructure/Δtotal curves for the model with values (l/d) =6 and (c/l) =0.15.

For modulus of elasticity ratio of 0.4 or higher the displacement ratio $\frac{\Delta_{Structure}}{\Delta_{total}}$ is approximately 1, which means that the assumption can be applied for any soil of modulus of elasticity ratio of 0.4 or higher, which includes the soil types of igneous rocks, limestone, sandstone, shale, dolomite and all the metamorphic rocks (Geotechdata, 2016).

After explaining the main assumptions and the limitations and boundaries, a certain zone is noticed that neither the rigid-structure flexible-soil assumption, nor the opposite assumption are applicable. This occurred because both the structure and the soil have significant displacements. This zone is obvious in Figure 3.4 and the boundaries of this zone are between the modulus of elasticity ratio of $4 * 10^{-4}$ to 0.4.

Because neither of the previous assumptions can be applied in this zone, it is important to find the displacement of both the structure and the soil, because ignoring one of these displacements may lead to unpredicted damages.

The physical meaning of the intersection in Figure 3.4, and that enlarged in Figure 3.5, is that for a certain modulus of elasticity ratio, both the soil and structure has an identical displacement, and each one shares half of the total displacement.

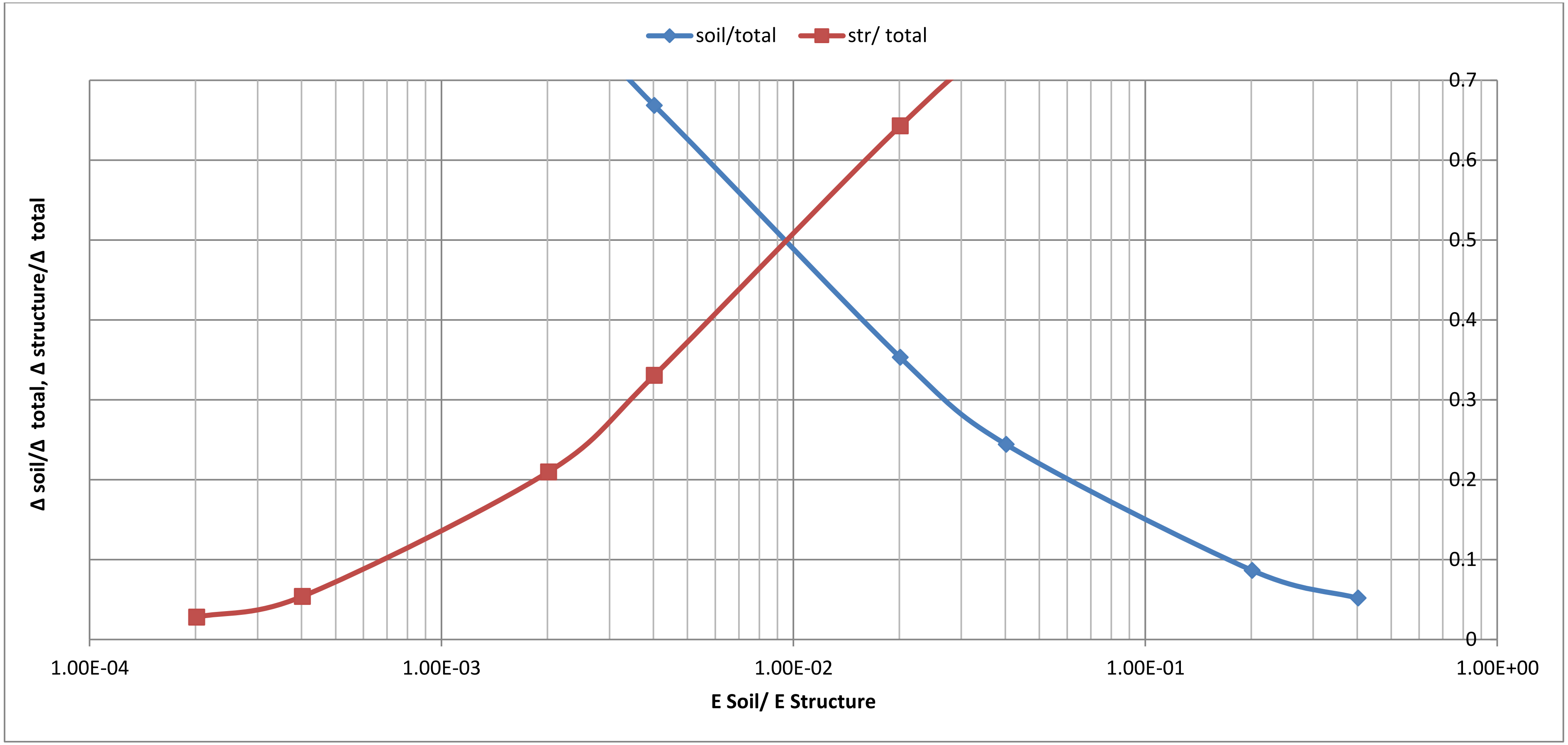

Figure 3.5: representative zoomed diagram shows Δsoil/Δtotal and Δstructure/Δtotal curves for the model with values (l/d) =6 and (c/l) =0.15.

3.6.2 Parametric study

After discussing the general behavior of the displacement ratios, it is important to study the change in the curves due to the change of $\left(\frac{l}{d}\right)$ and $\left(\frac{c}{l}\right)$ parameters. The ratios of soil displacement to total displacement and the displacement of structure to total displacement seem to give reliable logistic (S) curves when the modulus of elasticity ratio is scaled logarithmically, which will be very useful in measuring the differences and fitting the curves. Therefore, to achieve accurate and comparable data, the logarithmic value of the modulus of elasticity ratio will be used for the horizontal axis.

Figure 3.6 through Figure 3.11 show diagrams for $\frac{\Delta_{soil}}{\Delta_{total}}$ and $\frac{\Delta_{Structure}}{\Delta_{total}}$ curves respectively for various $\left(\frac{c}{l}\right)$ values, and Figure 3.12 shows diagrams for $\frac{\Delta_{soil}}{\Delta_{total}}$ curves for various values of $\left(\frac{l}{d}\right)$. It was noticed that all the curves have approximately the main starting and ending points in both types of curves, where for $\frac{\Delta_{soil}}{\Delta_{total}}$ the curves begin with value equals approximately 0 and end to a value of approximate value of 1, while the $\frac{\Delta_{Structure}}{\Delta_{total}}$ curves have the opposite behavior.

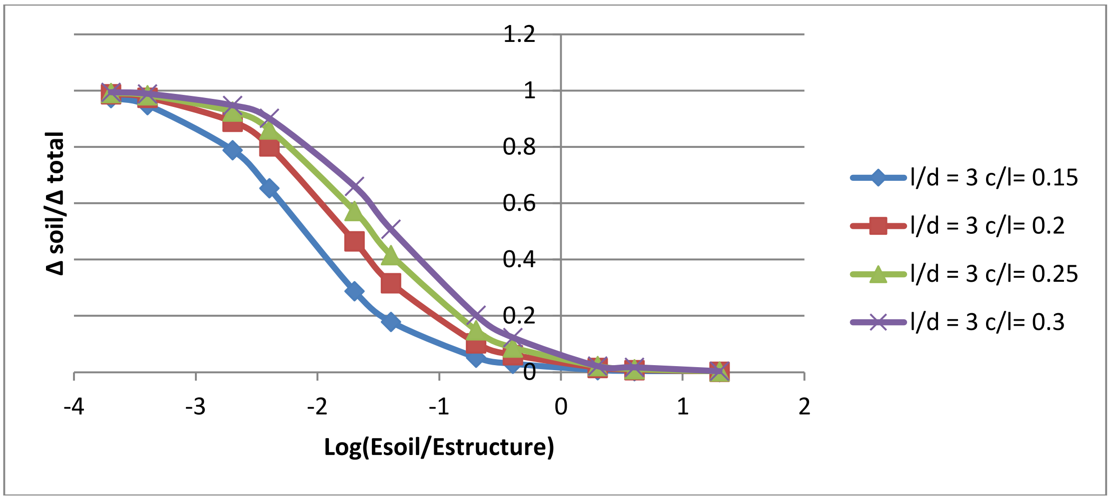

Figure 3.6: The change of $\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}}$ curves for $\left(\frac{l}{d}\right)$ value 3 and various $\left(\frac{c}{l}\right)$ values.

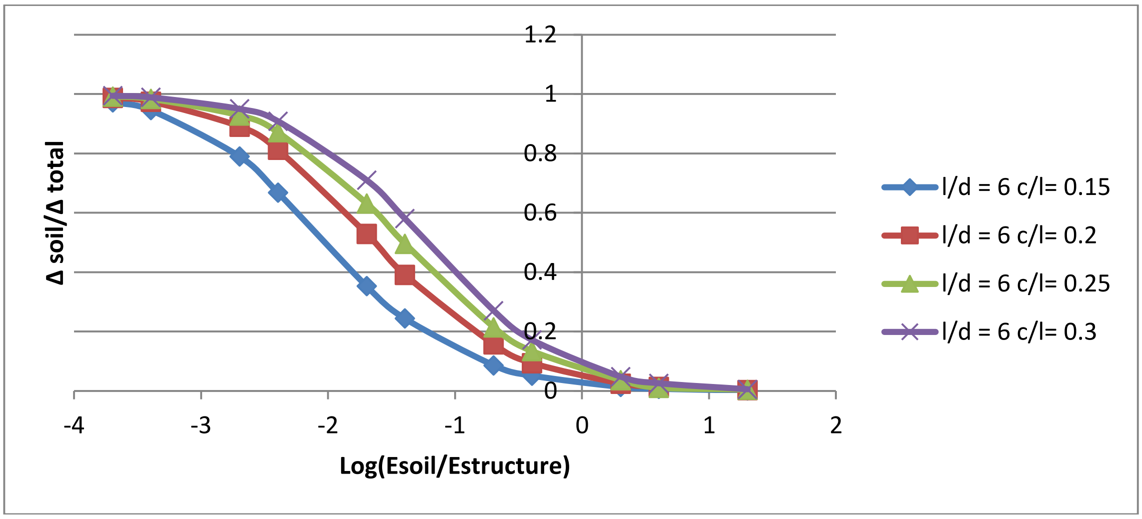

Figure 3.7: The change of $\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}}$ curves for $\left(\frac{l}{d}\right)$ value 6 and various $\left(\frac{c}{l}\right)$ values.

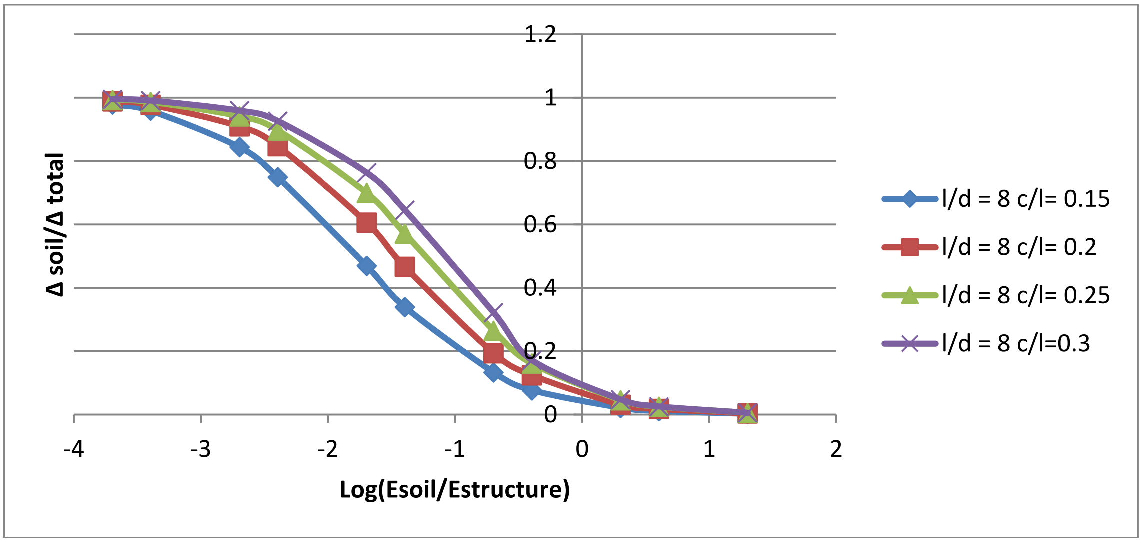

Figure 3.8: The change of $\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}}$ curves for $\left(\frac{l}{d}\right)$ value 8 and various $\left(\frac{c}{l}\right)$ values.

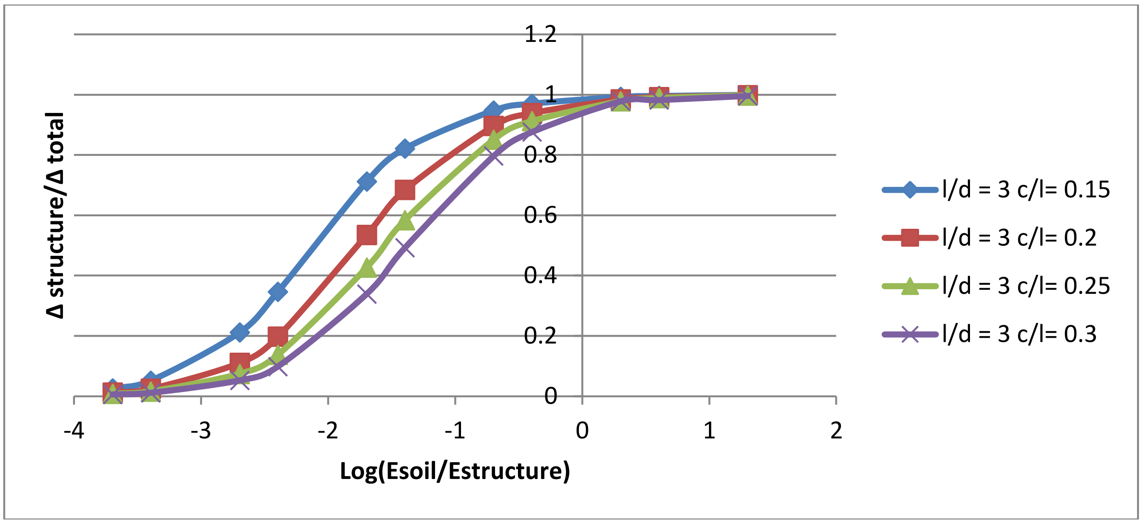

Figure 3.9: the change of $\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}}$ curves for value $\left(\frac{l}{d}\right)$ =3 and various $\left(\frac{c}{l}\right)$ values

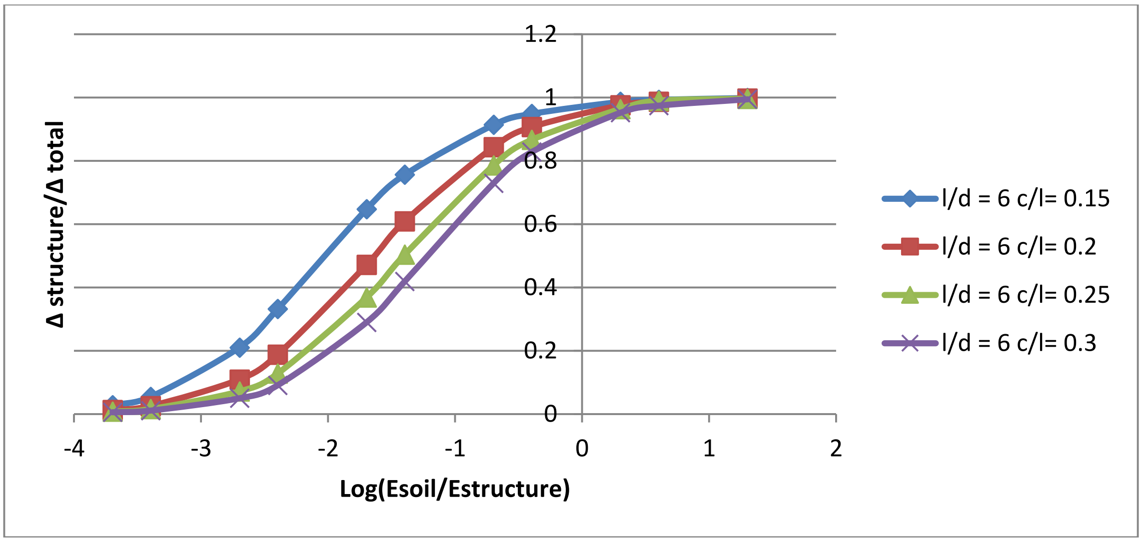

Figure 3.10: the change of $\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}}$ curves for value $\left(\frac{l}{d}\right)$ =6 and various $\left(\frac{c}{l}\right)$ values

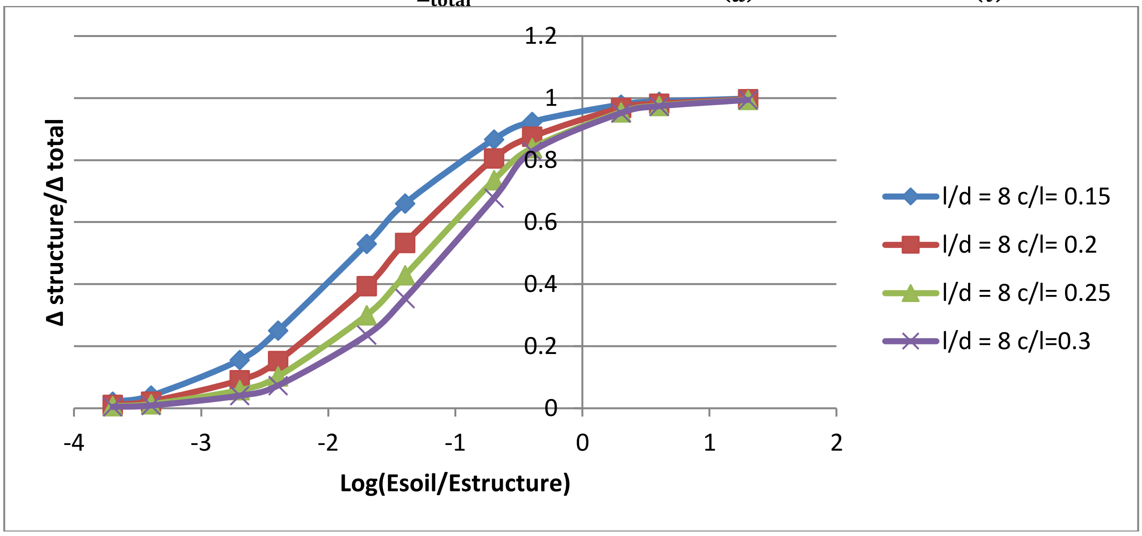

Figure 3.11: the change of $\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}}$ curves for value $\left(\frac{l}{d}\right)$ =8 and various $\left(\frac{c}{l}\right)$ values

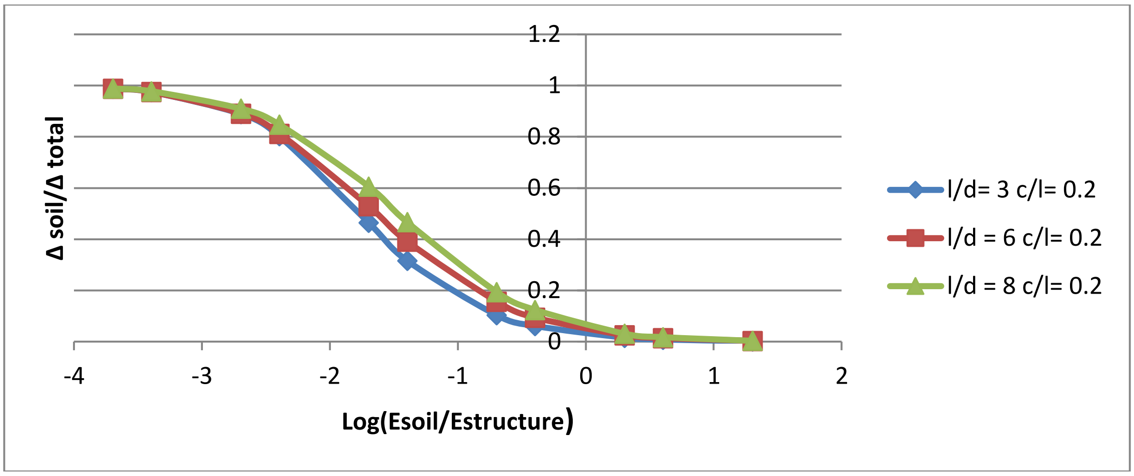

Figure 3.12: The change of $\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}}$ curves for $\left(\frac{c}{l}\right)$ value 0.2 and various $\left(\frac{l}{d}\right)$ values.

Moreover, it was noticed that for a certain modulus of elasticity ratio $\frac{E_{soil}}{E_{structure}}$, the displacement ratios change with the increase of $\left(\frac{c}{l}\right)$ and $\left(\frac{l}{d}\right)$ ratios. The change of the displacement ratio due to the increase of $\left(\frac{l}{d}\right)$ ratio is less significant than the change due to $\left(\frac{c}{l}\right)$ ratio, because the curves for different $\left(\frac{l}{d}\right)$ with same $\left(\frac{c}{l}\right)$ are successive with small changes, as can be seen in Figure 3.12, which shows the curves for different $\left(\frac{l}{d}\right)$ ratio for the same $\left(\frac{c}{l}\right)$ ratio.

This change in displacement ratios due to the change of parameters can be explained using basic mechanics of materials; the increase of $\left(\frac{l}{d}\right)$ ratio will cause a reduction of the footing depth, assuming the footing area being unchanged. Therefore, the footing rigidity will reduce, causing poor distribution of stress on the soil, which will increase the concentrated load effect around the column area because of the reduction of the footing rigidity, therefore increasing the soil settlement. Because the total displacement increases and the displacement of structure is approximately the same, the displacement ratios will change, where $\frac{\Delta_{Structure}}{\Delta_{total}}$ will decrease and $\frac{\Delta_{soil}}{\Delta_{total}}$ will increase.

On the other hand, increasing $\left(\frac{c}{l}\right)$ ratio will cause a significant increase in the $\frac{\Delta_{soil}}{\Delta_{total}}$ ratios. Any increase in the dimensions of the column will increase the column rigidity compared to the footing rigidity. The difference of the rigidity will lead to a flexible nature of the footing, which will lead to non uniform stress distribution and will reduce the footing effective area. This leads to magnify the stresses under the column area, developing partially concentrated effect on the soil which will increase the soil settlement.

To confirm the accuracy of the results and the validity of the physical meaning of the curves, it will be compared to the practical design process of footing. The design practice of the footing design states that when the stress of the structure affecting the soil is higher than the bearing capacity of the soil, the designer must increase the footing dimensions in order to increase the distribution area on the soil and thus the stress will decrease. This practice is simply matching the change of the $\left(\frac{c}{l}\right)$ ratio by decreasing it, which will give a lower $\frac{\Delta_{soil}}{\Delta_{total}}$ ratio, which means the physical meaning of the curves is acceptable. Although, decreasing $\left(\frac{c}{l}\right)$ values will increase $\left(\frac{l}{d}\right)$ values, thus increasing $\frac{\Delta_{soil}}{\Delta_{total}}$ a little, but because the effect of changing $\left(\frac{c}{l}\right)$ value is much higher than changing $\left(\frac{l}{d}\right)$ value, the settlement of soil will decrease.

3.6.3 Slope of curves and change of stress

The slope of the curves can be related to the rate of change of the stresses in soil and structure. The slopes of most curves are almost identical as can be seen in Figure 3.6 through Figure 3.11, this behavior is expected because the materials are assumed elastic. Thus, increasing the stresses will increase all the results with the same ratio, therefore the displacement ratios will not be affected. To assure that the slope of the curves will not change due to the change of stress values, a model for $\left(\frac{c}{l}\right)= 0.15$ and $\left(\frac{l}{d}\right)= 3$ assigned with stress of tripled value is simulated, which equals 18MPa, and the results are shown in Figure 3.13.

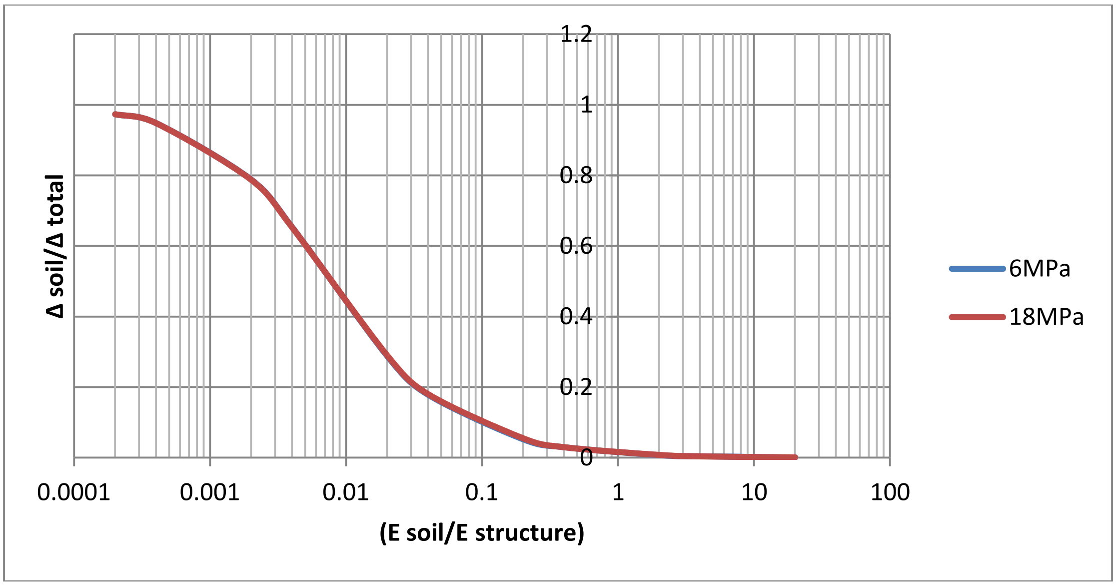

Figure 3.13: The change of $\frac{\Delta_{\mathbf{soil}}}{\Delta_{\mathbf{total}}}$ curves for varies values of stress, where σ is stress unit.

From Figure 3.13 it is obvious that both curves are identical with no change due to the change in stress. When the stress changes, the structure displacement changes along with the soil displacement, and the ratio between them does not change, this maintain constant ratios that are independent from the stress changing, in other words the stress is not a parameter in the displacement ratios. However, it has major role in calculating the actual displacement of the structure and the soil settlement.

3.7 Data fitting

After conducting the previous simulations, and finding that the results are reasonable and matching the common thinking about footing behavior and the general design practice of footings, it is important to have a general equation that can be used to predict the displacement for any similar structure with similar conditions.

As mentioned before, the curves give the logistic curve shape, which is "S" shape, which governed by the following equation (Weisstein, 2016):

$$f(x) = \frac{x_1}{1+e^{-k*(x-x_0)}} \tag{3.1}$$

Where:

x_1: the curve's maximum value.

k: steepness of the curve.

x_0: the x value of the Sigmoid's midpoint.

As was noticed from the previous figures, the curves maximum value is approximately 1, therefore, x_1 equals 1. The logarithmic value of the modulus of elasticity ratio will be used as the main dependent variable x-axis to simplify fitting the equation. In order to fit Equation 3.1 to the data, k and x_0 values were found for every simulated model. Then the diagrams of k and x_0 values were drawn and fitted into simple equations that govern these variables. After finding k and x_0 values for $\frac{\Delta_{soil}}{\Delta_{total}}$ values using the mathematical program Maple (Maplesoft, 2013), it was noticed that k gives values of the range 1.85 to 2.1. This confirms the previous conclusion that the curves approximately have the same slope. Therefore, to make the equation simple, k was assumed constant and equals 2. On the other hand, it was noticed that there are different values of x_0 according to the change of the values $\left(\frac{c}{l}\right)$ and $\left(\frac{l}{d}\right)$. This means an equation must be obtained in terms of these values to find the variable x_0. Figure 3.14 shows x_0 values for $\left(\frac{c}{l}\right)$ and $\left(\frac{l}{d}\right)$ values.

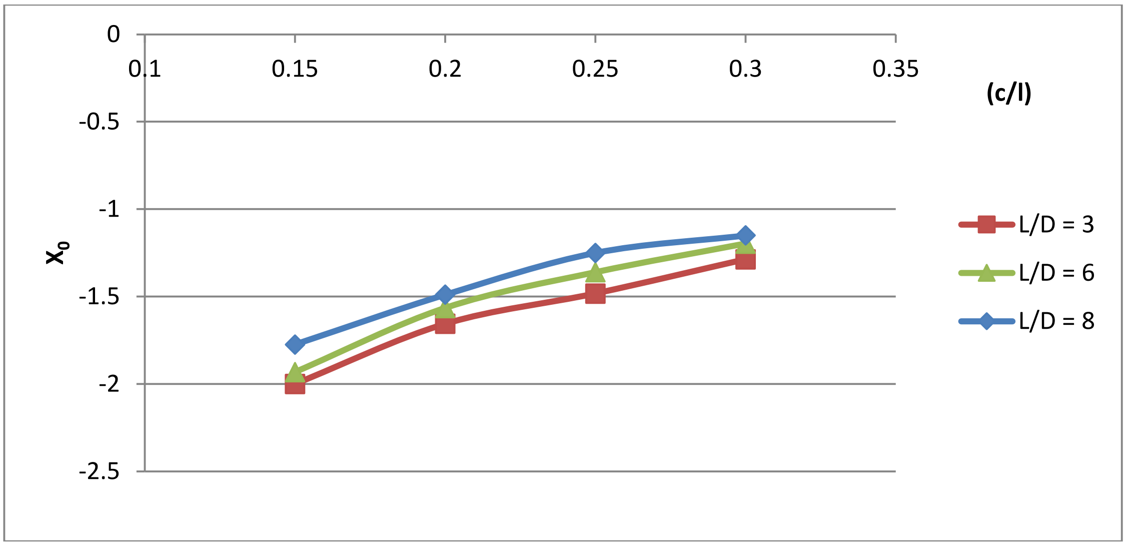

Figure 3.14: The change of $\mathbf{x_0}$ values with the changing of $\left(\frac{c}{l}\right)$ for $\left(\frac{l}{d}\right)$ values of 3, 6 and 8.

From Figure 3.14, it is noticed that the curves are governed by the natural logarithmic function $(\ln x)$. Therefore, it is concluded that the function governing x_0 is natural logarithmic function in terms of $\left(\frac{c}{l}\right)$ and $\left(\frac{l}{d}\right)$. As mentioned, the significant value is $\left(\frac{c}{l}\right)$, so the function concluded to be $x_0 = \ln(\alpha) * \ln\left(\frac{c}{l}\right)$, where α is another variable in terms of $\left(\frac{l}{d}\right)$. By dividing the concluded results of x_0 from the previous experiments by the value of $\ln\left(\frac{c}{l}\right)$, α values are found and drawn in a curve in term of $\left(\frac{l}{d}\right)$, and the equation were found as a polynomial function, see Figure 3.15.

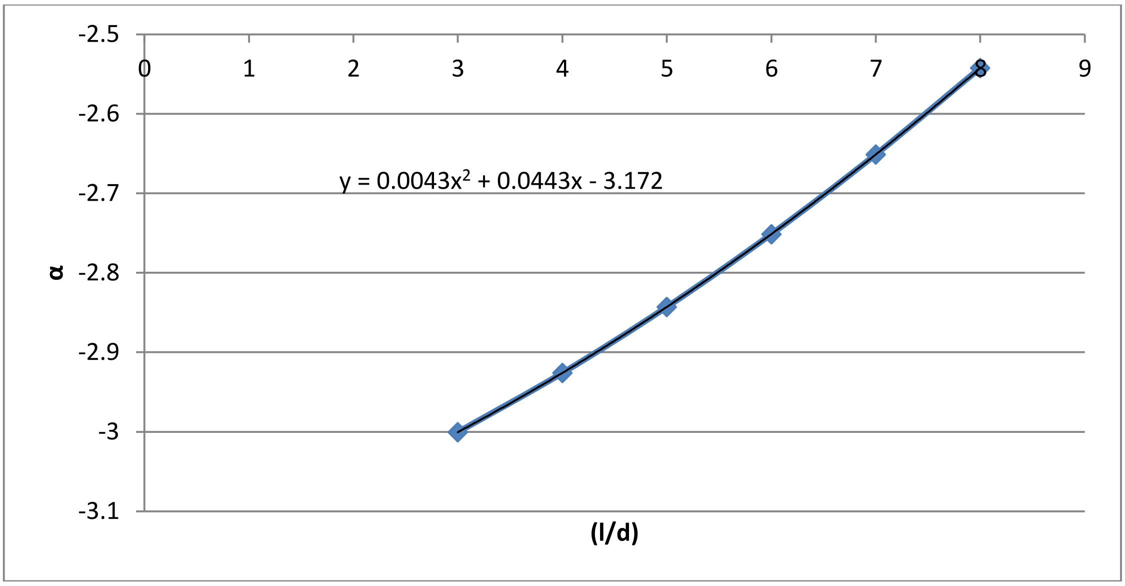

Figure 3.15: A curve representing the relationship between $\left(\frac{l}{d}\right)$ and the variable $\boldsymbol{\alpha}$.

The equations that govern the displacement ratios are:

$$\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}} = \frac{1}{1+e^{2*(\log(\text{Sr})+x_0)}} \tag{3.2}$$

$$\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}} = 1 - \frac{1}{1+e^{2*(\log(\text{Sr})+x_0)}} \tag{3.3}$$

Where

$$\text{Sr} = \frac{\text{E}_{\text{soil}}}{\text{E}_{\text{structure}}} \tag{3.4}$$

$$x_0 = \ln(\alpha) * \ln\left(\frac{c}{l}\right) \tag{3.5}$$

$$\alpha = 0.0043\left(\frac{l}{d}\right)^2 + 0.0443\left(\frac{l}{d}\right) - 3.1720 \tag{3.6}$$

3.7.1 Equation verification

To assure the accuracy of these equations, it is tested on a model with $\left(\frac{l}{d}\right) = 4$ and $\left(\frac{c}{l}\right) = 0.2$. The results are shown in Figure 3.16, Figure 3.17, and **Error! Reference source not found.**.

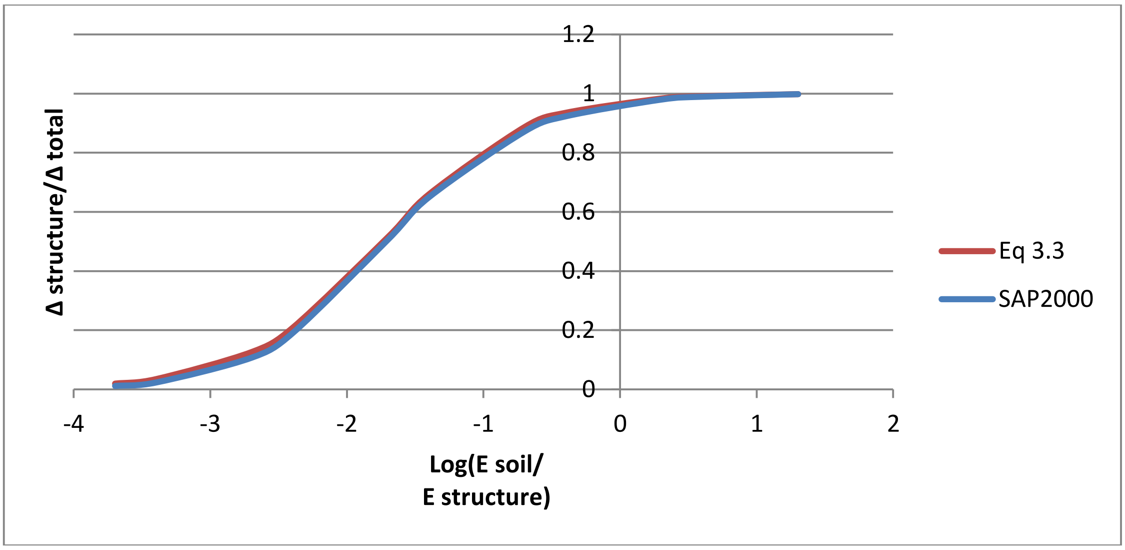

Figure 3.16: Comparison between the finite element results and the equation 3.3 results for $\frac{\Delta_{\mathbf{structure}}}{\Delta_{\mathbf{total}}}$ curve.

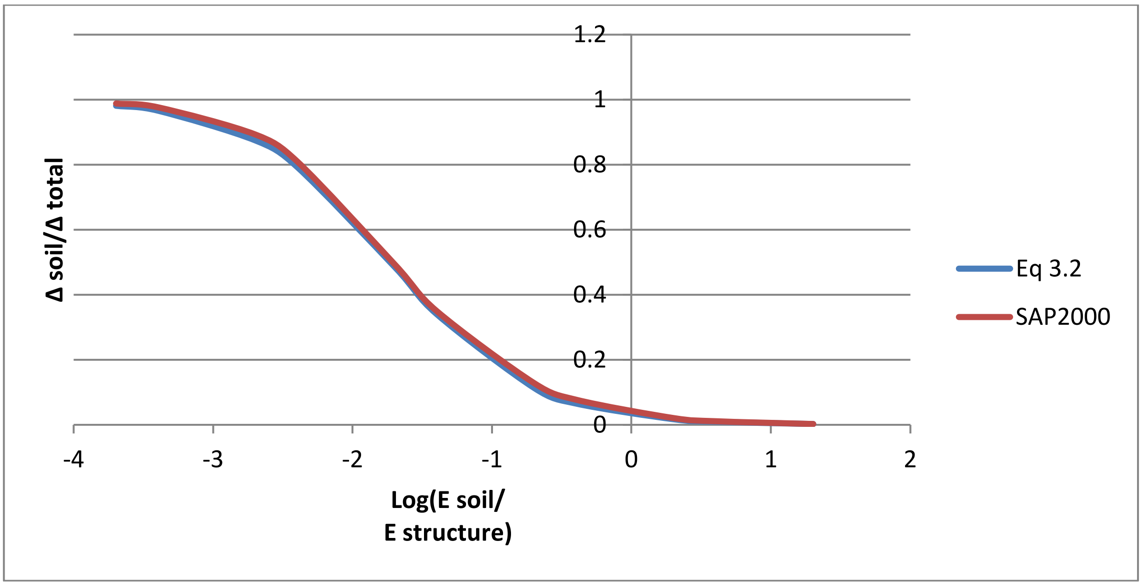

Figure 3.17: Comparison between the finite element results and the equation 3.2 results for $\frac{\Delta_{\mathbf{soil}}}{\Delta_{\mathbf{total}}}$ curve.

Figure 3.16 and Figure 3.17 show that the curves from the numerical solution are almost identical to those generated by the equation. Also, **Error! Reference source not found.** shows the percentage of error between the results of the finite element analysis and the results of the equation, taking the finite element results as the reference.

Error! Reference source not found. indicates having an acceptable percent of error with maximum value of approximately 16% for $\frac{\Delta_{soil}}{\Delta_{total}}$ and $\frac{\Delta_{sructure}}{\Delta_{total}}$ for the in between zone– the zone placed in the modulus of elasticity ratio range of $4 * 10^{-4}$ to 0.4.

Table 3.3: Difference between the finite element results and the equation results for a model with $\left(\frac{l}{d}\right) = 4$ and $\left(\frac{c}{l}\right) = 0.2$.

l/d	4		A	2.926	x_o	1.72795				
c/l	0.2				K	2				
Log(Esoil/ Estructure)	E soil/ E structure	Δ total	Δ soil	Δ Structure	Δsoil /Δtotal	ΔStructure / Δ total	Eq (3.3) Δsoil/ Δ total	% error Δsoil/ Δtotal	Eq (3.4) ΔStructure / Δtotal	% error ΔStructure /Δ total
-3.70	2.01E-04	61.30	60.55	0.75	0.99	0.01	0.98	0.70	0.02	56.80
-3.40	4.02E-04	31.10	30.35	0.75	0.98	0.02	0.97	1.07	0.03	43.52
-2.70	2.01E-03	6.94	6.19	0.75	0.89	0.11	0.87	2.04	0.13	16.88
-2.40	4.02E-03	3.91	3.16	0.75	0.81	0.19	0.79	2.12	0.21	8.97
-1.70	2.01E-02	1.47	0.73	0.75	0.49	0.51	0.48	1.66	0.52	1.61
-1.40	4.02E-02	1.15	0.40	0.75	0.35	0.65	0.34	2.71	0.66	1.45
-0.70	2.01E-01	0.86	0.11	0.75	0.13	0.87	0.11	12.32	0.89	1.82
-0.40	4.02E-01	0.81	0.06	0.75	0.08	0.92	0.07	16.57	0.93	1.40

On the other hand, the modulus of elasticity ratio of $4 * 10^{-4}$ or less have a high percentage of error for $\frac{\Delta_{sructure}}{\Delta_{total}}$, which can be explained by noticing how small the values are, which makes any small amount of change in the value significant as percent of error. Moreover, the zones of modulus of elasticity of $4 * 10^{-4}$ or less and 0.4 or more have special cases because of the total applicability of the rigidity assumptions, which make the equation applicability on these zones not significant.

Furthermore, to have more confidence in the equations, another verification will be used, where random values of $\left(\frac{l}{d}\right)$ and $\left(\frac{c}{l}\right)$ were chosen for certain Sr, and the results were calculated by SAP2000 and by the equation. The results were compared by having the SAP2000 results at the horizontal axis and the equation's results at the vertical axis. Figure 3.18 and Figure 3.19 show the compared results diagrams.

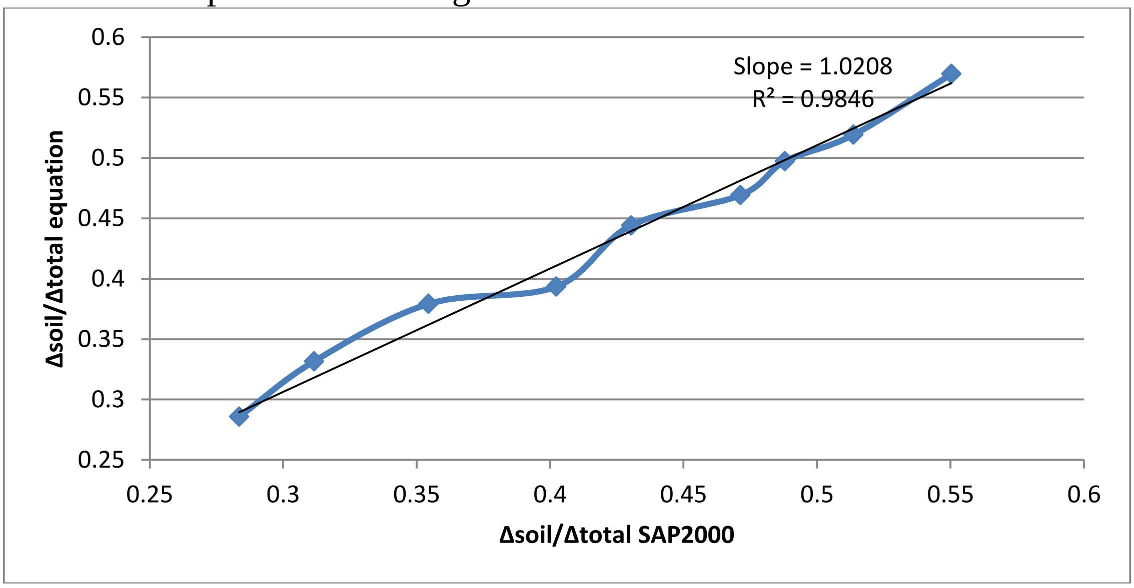

Figure 3.18: $\frac{\Delta_{soil}}{\Delta_{total}}$ from SAP2000 versus $\frac{\Delta_{soil}}{\Delta_{total}}$ from Equation 3.2.

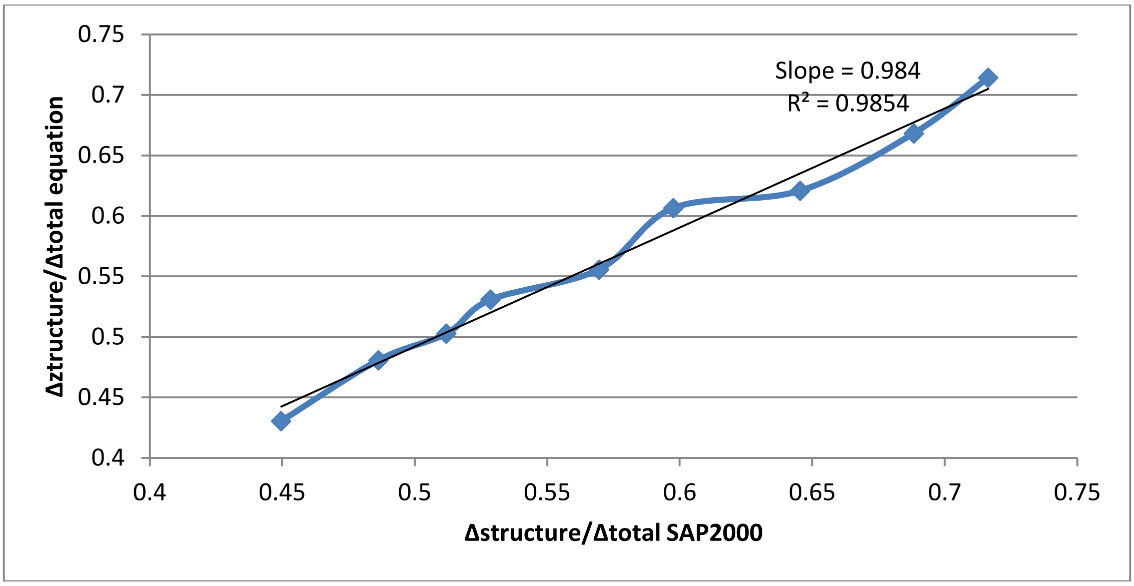

Figure 3.19: $\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}}$ from SAP2000 versus $\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}}$ from Equation 3.3.

The calculated slope values from the curves can be approximated to 1, which is considered an acceptable value.

To test the applicability of the equations for the upper and lower limits of the modulus of elasticity ratios, the following calculations have been conducted.

For the flexible structure rigid soil assumption, the Sr value is ∞, and the structure must participate in100% of the total displacement.

$$\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}} = \frac{1}{1 + e^{2*(\log\infty + x_0)}} = 0$$

$$\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}} = 1 - 0 = 1$$

On the other hand, for the rigid structure flexible soil assumption, the Sr equal 0 which gives logarithmic value of $-\infty$, therefore exponential value of 0.

$$\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}} = \frac{1}{1 + e^{2*(\log(0) + x_0)}} = 1$$

$$\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}} = 1 - 1 = 0$$

After assuring that the equations give acceptable results for the upper limit and the lower limit, and that the equations' results are approximate to those from the finite element tool, it can be stated that the equations are acceptable and can be used for cases with similar assumptions.

3.7.2 Height of column

To take all aspects into consideration, the height of the column must be taken as an effective parameter. All of the previous tests were done having a constant height of column of 3 meters, that makes the previous equation a special equation that do not explain the displacement ratios for any other structure with different height.

Before deriving the general equation, the displacement of the structure must be explained by mechanics of materials. The main displacement in the column came from the vertical loads affecting it, which means an axial deformation, which is governed by the equation:

$$\Delta = \frac{Ph}{EA} \qquad (3.7)$$

Where

P : the axial load.

h : the length of the column.

E : the modulus of elasticity.

A : the area of the column.

Thus, the displacement is increasing linearly with the increase of the column height, and the general equation becomes:

$$\frac{\Delta_{structure}}{\Delta_{total}}_{h} = \frac{\left(\frac{\Delta_{structure}}{\Delta_{total}}_{3m}\right) * \frac{h}{3}}{\left(\frac{\Delta_{structure}}{\Delta_{total}}_{3m}\right) * \frac{h}{3} + \left(\frac{\Delta_{soil}}{\Delta_{total_{3m}}}\right)} \qquad (3.8)$$

$$\frac{\Delta_{soil}}{\Delta_{total_h}} = 1 - \frac{\Delta_{structure}}{\Delta_{total}}_{h} \qquad (3.9)$$

Where

$\frac{\Delta_{structure}}{\Delta_{total}}_{h}$: the displacement of structure ratio for *h* meters length

$\frac{\Delta_{soil}}{\Delta_{total_h}}$: the displacement of soil ratio for *h* meters length

$\frac{\Delta_{structure}}{\Delta_{total}}_{3m}$: the displacement of structure ratio for *3* meters length.

$\frac{\Delta_{soil}}{\Delta_{total_{3m}}}$: the displacement of soil ratio for *3* meters length.

Figure 3.20 and Figure 3.21 show the comparison between the results from the finite element program and from the equations 3.8 and 3.9 for a model with column height of 4.5m. It is obvious that both curves in both diagrams are identical, thus the equation is acceptable because it satisfies the accepted accuracy.

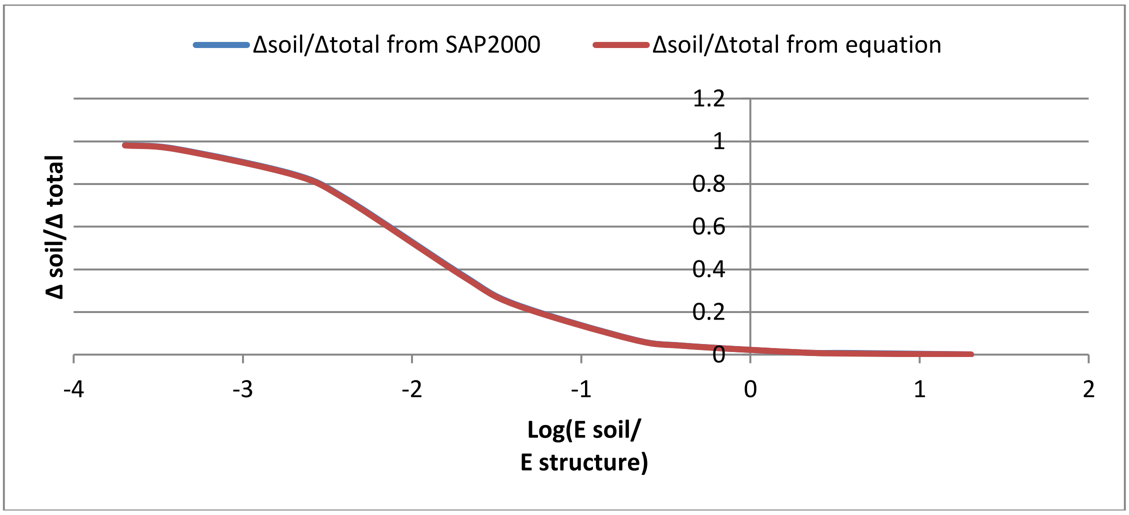

Figure 3.20: Comparison curves between the finite element results and equation 3.8 results for the $\frac{\Delta_{soil}}{\Delta_{total}}$ values for a model with height of 1.5 *l* of the previous experiments.

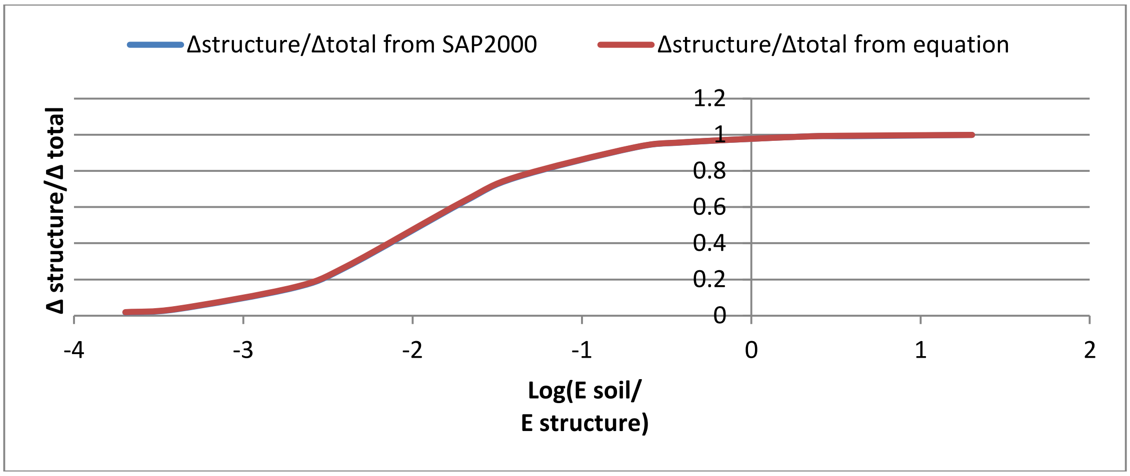

Figure 3.21: Comparison curves between the finite element results and equation 3.9 results for the $\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}}$ values for a model with height of 1.5 *l* of the previous experiments.

3.7.3 Limitations of the equation

After creating many models in SAP2000 and testing them, it was noticed that Equations 3.2 and 3.3 cannot predict the behavior and the displacement ratios for the structures with $\left(\frac{c}{l}\right)$ ratios of less than 0.15. The explanation of this is the effect of the difference of the area of the footing to the area of the column and the associated rigidity difference, where decreasing $\left(\frac{c}{l}\right)$ ratio would not be practical as most of the footing area would not be as effective as if the $\left(\frac{c}{l}\right)$ value is higher. This occurred because the column acts as a concentrated load in a small area.

Another case is that when the equation failed to explain the results when $\left(\frac{l}{d}\right)$ has value more than 8. Increasing $\left(\frac{l}{d}\right)$ ratio will cause a sever reduction in the rigidity of the footing, which gives it flexible behavior compared with the column's rigidity, this affect the effective area of the footing by reducing it, thus the displacement ratios will be changed significantly because the effective footing dimensions has changed.

4 Two dimensional frames

4.1 Introduction

All the previous models are simple models of very simple structure. However, the situation is more complex if applied on frames. Small errors are expected if the soil beneath the frame is considered uniform of the same soil properties, because each column will likely influence only the soil beneath it. On the other hand, it is predicted in this research to have significant errors for frame with different types of soils with significant modulus of elasticity variation under the columns. The different properties of soil will cause less settlement than that calculated for the weak soil, and more settlement for the stronger soil. This might occur because the unequal settlements of soil, which will cause a redistribution of load transfer in the columns. Thus, it is important to understand the effect of the connected members on the displacement ratios and the fitted equations.

4.2 Verification

To assure the accuracy of the results from the three dimensional multi-nodded elements model, the three dimensional multi-nodded model will be compared with results of acceptable accuracy. Analogical comparison is conducted with a frame model simulated in SAP2000 as a reference, using 1D elements. The results from the 1D frame model is considered accurate through sensitivity study on the model similar to the one done for the previous model. To achieve the same conditions for the both models, some assumptions are made; The frame used in the comparison is the two-span frame. In the three dimensional multi nodded frame, the soil is considered rigid body while the material of structure and footings is concrete. On the other hand, the two dimensional frame is restrained with fixed joints. Figure 4.1 shows the moment results from the three dimensional multi-nodded model versus the results from the one-dimensional frame. It is obvious from the figure that the slope value equals 1.12 with error within the acceptable range, while the coefficient of determination value is approximately 1. This means the results are acceptable, and the three dimensional multi-nodded frame models can be used to find the reactions for the frame in soil-structure interaction conditions.

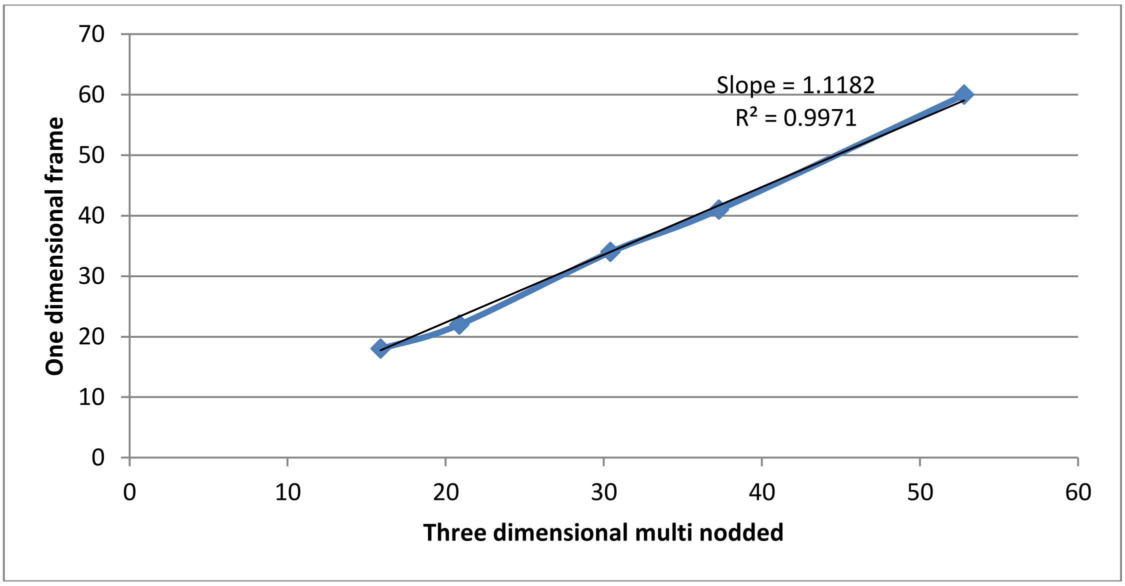

Figure 4.1: Comparison between the three dimensional multi nodded frame and the two dimensional frame.

4.3 Structural and material models

To find the change in the displacement ratios when the structure has higher complexity, a simple frame will be adopted as the testing model. The one span frame will not have settlement of the soil comparable with the results of the one column model previously discussed. The reason is that the beam-column connection has small stiffness, which will not affect the displacements of soil and structure. Therefore, to estimate the effect of the beam on the settlement, one-story two equal spans frame model with identical footings and columns dimensions and heights will be used.

In addition to the parameters mentioned in Section 3.2, other parameters are required to describe the frame model. These parameters are listed as follows:

- Number of spans.
- The clear length of spans. L_{nb}
- Dimensions of the beam.
- The variation of the modulus of elasticity of soil under footings.

Figure 4.2 shows a sketch for two-spans frame, with the parameters used in simulation.

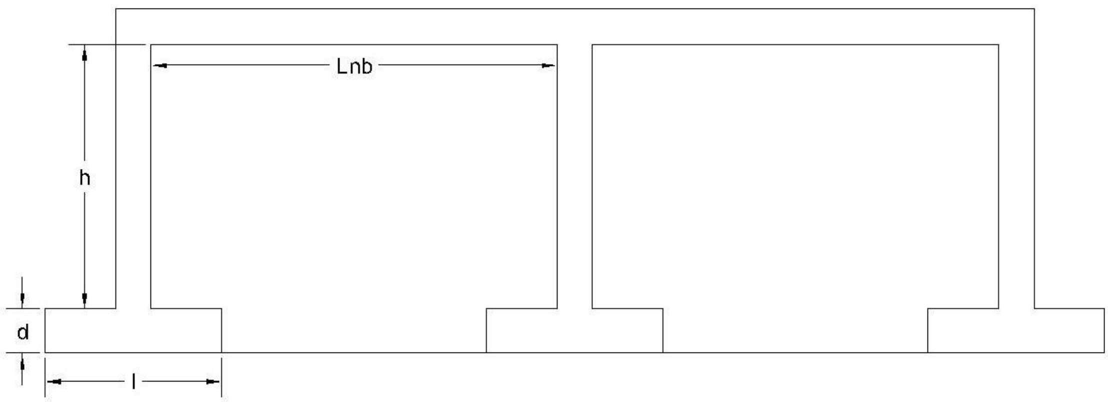

Figure 4.2: The frame used as model with the parameters used in the simulation.

Unlike the simple model which has distributed stress directly on the top of the column, the stresses are considered distributed on the beam in the frame model with values approximately equivalent to the service loads of 125kN/m^2. This value was chosen according to previous experience in practical ranges for the service loads. The load was applied as area pressure on the top face of the three dimensional beam elements. The height of the columns is considered constant and equals 3m.

Two types of soil are used, soil with variable modulus of elasticity under the middle footing, with modulus of elasticity (E_1) of values within the range of 5MPa to 10000 MPa. The other type has a constant high modulus of elasticity of E_2=25000MPa. As for the material properties and assumptions, the materials are assumed to be linear, elastic, homogenous and isotropic as were mentioned in Section 3.3.

Steel reinforcing is ignored in the frame model, for the same reasons discussed in Section 3.4.

To have acceptable accuracy, the beams, columns and footings are divided into fine mesh, in order to obtain more accurate data for the stress and displacement. Figure 4.3 shows cross section of the frame from the finite elements software SAP2000. On the other hand, the soil is meshed so that the volume of the soil element decreases when approaching the structure. This has been done in order to reduce the duration of analysis, while maintaining the accuracy of the results values (the mesh sensitivity was discussed in Section 2.4).

4.4 Basic assumptions

Because the program SAP2000 treats the three dimensional multi nodded element as total linear, elastic, homogenous and isotropic object, the soil has tension in certain places between the columns due to high settlements. Thus, to minimize the tension volumes effect, certain cuts have been done to the soil volume where the force is a tensile force. The final cross section of the model is shown in Figure 4.3 where the cut volumes are obvious between the footings. The stress distribution that was explained in Chapter 2 is taken into consideration to assure that no excessive volumes are removed. The shear stress effect expected to affect the surface beneath the cut volume due to the loss of these volumes has been noticed, and found to be very small and ignorable.

In order to normalize the results, the parameters are normalized into ratios. The ratios used in Chapter 3, which are: the ratio of length of footing side to depth of footing $\left(\frac{l}{d}\right)$, length of column side to length of footing side $\left(\frac{c}{l}\right)$ and the ratio of the soil modulus of elasticity to the concrete modulus of elasticity$\frac{E_{soil}}{E_{structure}}$. Moreover, additional ratios are used in the frame analysis in order to reduce the number of variables. There are the relative stiffness ratio (G), which is the ratio of column stiffness to beam stiffness and clarified in Equation 4.1, and the variable soil to constant soil modulus of elasticity ratio $\frac{E_1}{E_2}$.

$$G = \frac{EI_c/h}{nEI_b/l_{nb}} \qquad (4.1)$$

Where:

E: modulus of elasticity of the material used in structure.

I_c: moment of inertia of the column.

I_b: moment of inertia of the beam.

n: number of beams.

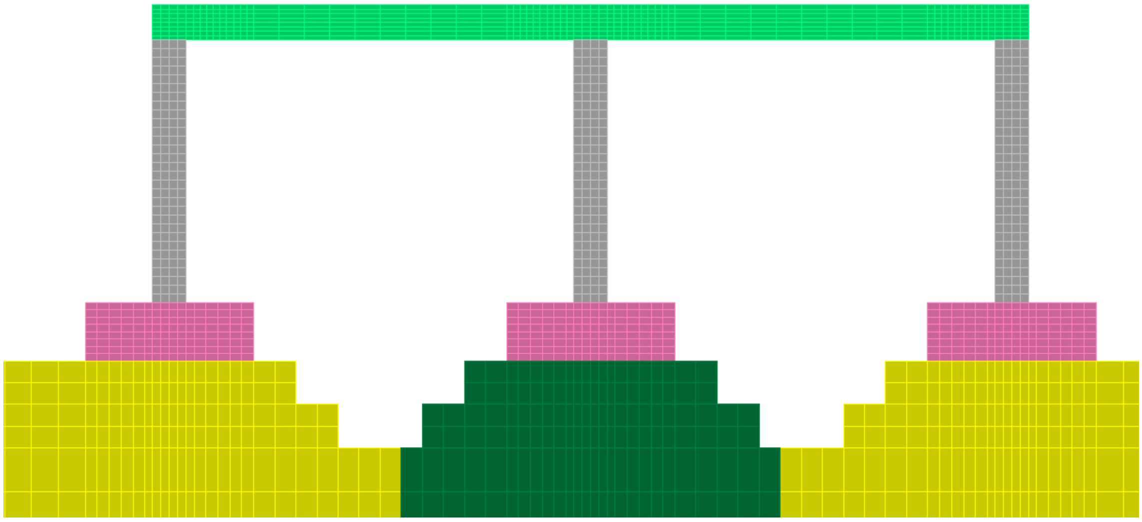

Figure 4.3: Section view in 3D model showing the mesh dimensions and the tension volume cuts.

4.5 Procedure

To achieve the maximum possible changes that may occur to the settlements, the soil is divided into two main categories; The middle soil is considered variable soil with modulus of elasticity (E_1) that varies between 5MPa to 10000MPa. The soil beneath the edge columns is considered constant with modulus of elasticity (E_2) equals to 25000MPa. Then, the modulus of elasticity ratio of the variable soil to the constant soil $\frac{E_1}{E_2}$ is calculated. These values are chosen to achieve the variability of the ratio $\frac{E_1}{E_2}$, where it covers small ratios as well as high ratios. Table 1 shows the chosen moduli of elasticity used in the finite element model.

Table 4.1: The moduli of elasticity and modulus of elasticity ratios used in the frame models

E_1	5	10	50	500	5000	10000
E_1/E_2	0.0002	0.0004	0.002	0.02	0.2	0.4

As was mentioned earlier in Chapter 3, $\left(\frac{c}{l}\right)$ and $\left(\frac{l}{d}\right)$ ratios are significant ratios that affect the displacement ratios. Therefore, to understand the effect of the frame on the displacement ratios, many models with several $\left(\frac{c}{l}\right)$ and $\left(\frac{l}{d}\right)$ ratios are simulated on the finite elements software, and the results for each model are tabulated and compared with the calculated results from Equation 3.2 and Equation 3.3.

To take the effect of the beam dimensions on the settlement of soil into consideration, the depth of the beam is considered variable while the width is considered constant. The depth of the beam is chosen because it has the significant effect on the vertical moment of inertia of the beam. The variability of the beam dimensions includes: beams with the same depth as the column, half the depth of the column and twice the depth of the column, in order to achieve wide range of beam dimension ratios. Because the depth of beam is changing, and to have a reference for the structure height and displacement, the displacement of structure is taken from the top of the column, which includes the displacement of the column and the footing only, following the same procedure that was mentioned in Section 3.5.

Likewise, to discuss the effect of the length of the span on the soil settlements, several models are simulated with the span length as a variable and the other parameters are assumed constants. The results from these models are tabulated and the effect of the span length will be discussed in the next sections.

However, the effects of the beam dimensions and the length of spans are included by the column stiffness to beam stiffness ratio (G). Therefore, these parameters will be covered in the discussion of the G ratio effect.

4.6 Results and discussion

After the models were analyzed, the displacements of the structure, the settlements of soil and the total displacements are found and calculated for the middle column, in order to find the displacement ratios $\frac{\Delta_{soil}}{\Delta_{total}}$ and $\frac{\Delta_{Structure}}{\Delta_{total}}$ and draw the curves. The curves obtained from the finite elements method are compared with the curves formed using Equation 3.2. Because the displacement ratios are interdependent, only $\frac{\Delta_{soil}}{\Delta_{total}}$ will be discussed. As mentioned earlier, the parameters are normalized into ratios; these ratios will act as a reference in the discussion of the effect of the frame on the soil settlements. Thus, the raw results will be presented for each aspect separately, and the noticed points will be discussed for each section.

4.6.1 Effect of column and footing parameters

In Chapter 3 the effect of $\left(\frac{c}{l}\right)$ ratio and $\left(\frac{l}{d}\right)$ ratio on the displacement ratios for simple structure was discussed. However, the effect of these ratios on the simple frame must be discussed to find out the changes occurred due to the change of these ratios.

To discuss the effect of $\left(\frac{c}{l}\right)$ ratio, Table 4.2 and

Table 4.3 show the tabulated results for representative models with the same $\left(\frac{l}{d}\right)$ and G ratios and different $\left(\frac{c}{l}\right)$ ratios. Furthermore, Figure 4.4 shows the change of displacement ratio curves $\frac{\Delta_{soil}}{\Delta_{total}}$ while changing $\left(\frac{c}{l}\right)$ ratio. Figure 4.5 and Figure 4.6 show the comparison of the displacement ratio curves $\frac{\Delta_{soil}}{\Delta_{total}}$ between the frame finite elements results and the results calculated from Equation 3.2. It is obvious from Figure 4.4 that increasing $\left(\frac{c}{l}\right)$ ratio will increase the soil displacement ratio, which is identical to the simple structure behavior that was mentioned in Chapter 3, and the reason of this behavior was discussed in Section 3.6.2.

Figure 4.5 and Figure 4.6 represent the finite element soil settlement ratio curve and those calculated from Equation 3.2, which from, it is noticed that the soil displacement ratios are not affected by the addition of the beams and the additional columns. To assure the accuracy of the results, Figure 4.7 shows a diagram of the finite element displacement ratio values versus the displacement ratio values calculated from Equation 3.2. It is obvious that the diagram is approximately linear, with slope value and R^2 value of approximately 1, which means that the values of both methods are approximately equal. On the other hand, as stated before, changing $\left(\frac{l}{d}\right)$ ratio while considering $\left(\frac{c}{l}\right)$ ratio as constant will change the displacement ratios in the simple model. To assure that Equation 3.2 is applicable for the changing of $\left(\frac{l}{d}\right)$ ratio, the frame will be tested by comparing the results from many models with different $\left(\frac{l}{d}\right)$ ratios for the same $\left(\frac{c}{l}\right)$ ratio and G ratio.

From Figure 4.8 it is obvious that increasing $\left(\frac{l}{d}\right)$ ratio increases the soil settlement ratio, which is the same as the simple model. While Figure 4.9 shows a comparison between the finite elements results and Equation 3.2, where no significant errors are noticed. Furthermore, Figure 4.10 shows the soil settlement ratio for the finite elements results versus Equation 3.2. It is obvious that the slope and R^2 values approximately equal 1, which is an indication that the results are acceptable.

Table 4.2: The displacements and the displacement ratios from finite elements method for frame with c/l ratio equals 0.2 and l/d value equals 3 and G value of 0.833.

E Soil	Δ total mm	Δ Soil mm	Δ Structure mm	E soil/ E Structure	Δ soil/ Δ total	Δ Structure / Δ total
5	10.5	10.35	0.15	2.01E-04	0.9857	0.0143
10	6.22	6.04	0.18	4.02E-04	0.9711	0.0289
50	1.61	1.41	0.2	2.01E-03	0.8758	0.1242
500	0.371	0.161	0.21	2.01E-02	0.4340	0.5660
5000	0.236	0.024	0.212	2.01E-01	0.1017	0.8983
10000	0.227	1.40E-02	0.213	4.02E-01	0.0617	0.9383

Table 4.3: The displacements and the displacement ratios from finite elements method for frame with c/l ratio equals 0.25 and l/d value equals 3 and G value of 0.833.

E Soil	Δ total mm	Δ Soil mm	Δ Structure mm	E soil/ E Structure	Δ soil/ Δ total	Δ Structure / Δ total
5	12.6	12.5	0.1	2.01E-04	0.9921	0.0079
10	6.32	6.2	0.12	4.02E-04	0.9810	0.0190
50	1.86	1.71	0.15	2.01E-03	0.9194	0.0806
500	0.37	0.2	0.17	2.01E-02	0.5405	0.4595
5000	0.198	0.029	0.169	2.01E-01	0.1465	0.8535
10000	0.187	0.017	0.17	4.02E-01	0.0909	0.9091

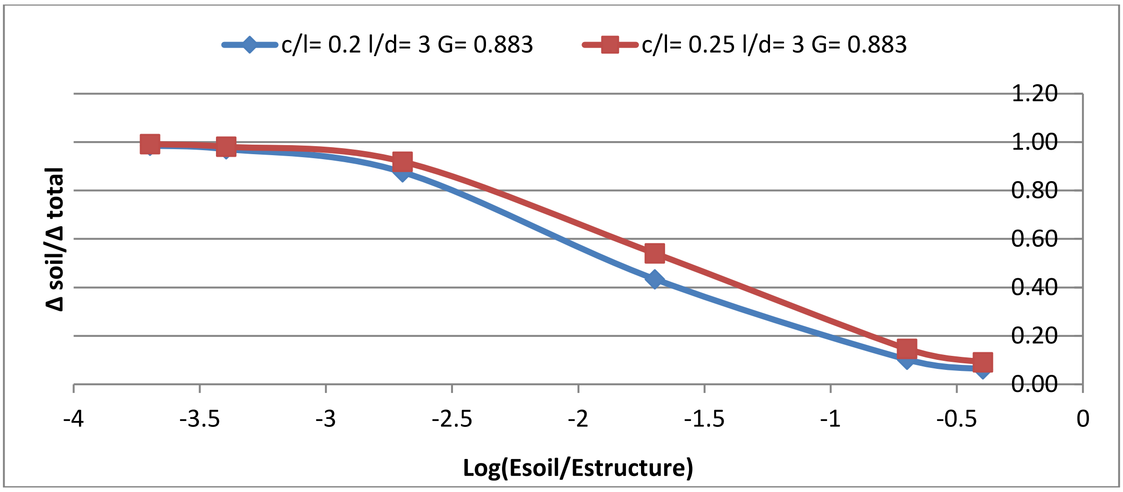

Figure 4.4: The change in the curves with the change of $\frac{c}{l}$ ratio, considering the other variables are constants.

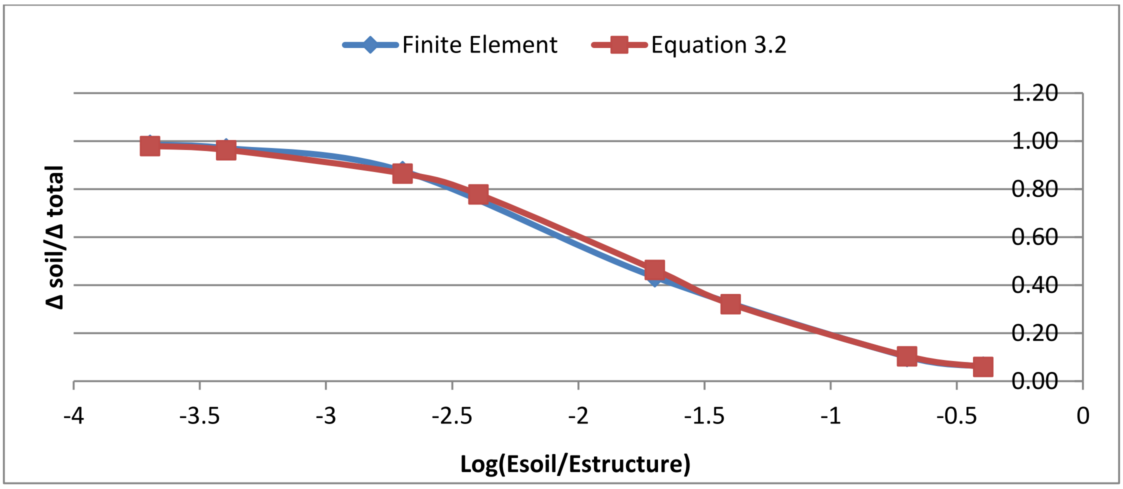

Figure 4.5: Comparison between the finite elements method and Equation 3.2 results for frame with $\frac{c}{l}$ ratio equals 0.2 and $\frac{l}{d}$ value equals 3 and G value of 0.833.

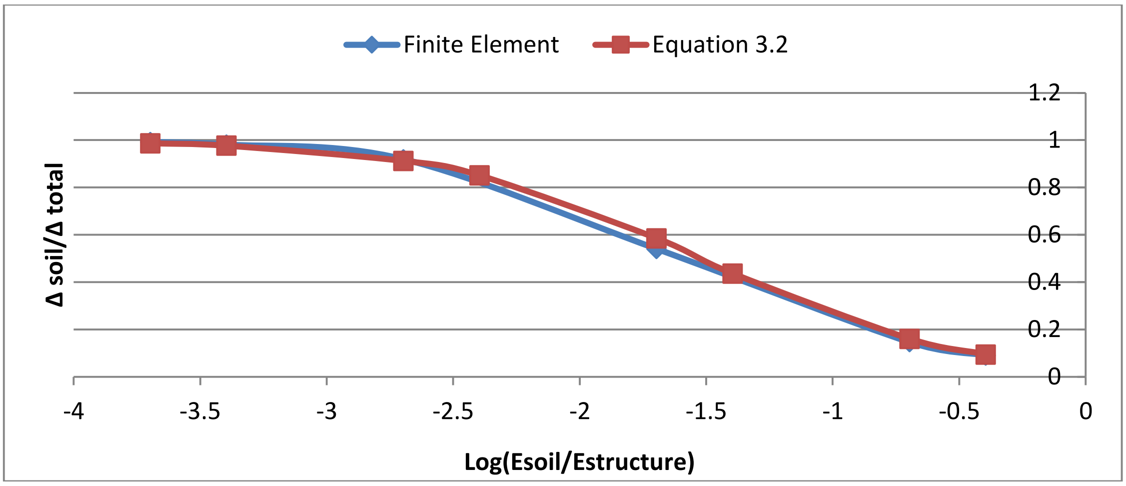

Figure 4.6: Comparison between the finite elements method and Equation 3.2 results for frame with $\frac{c}{l}$ ratio equals 0.25 and $\frac{l}{d}$ value equals 3 and G value of 0.833.

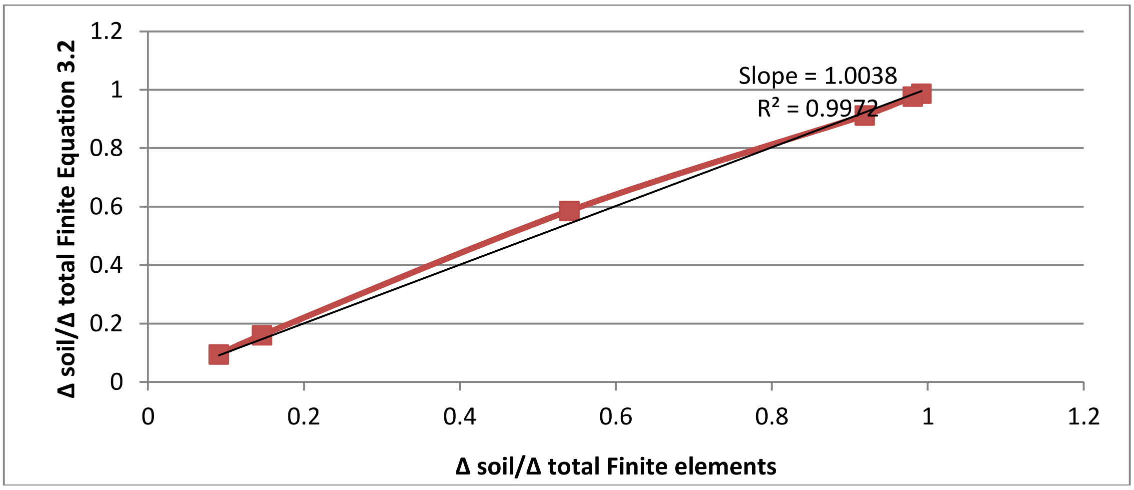

Figure 4.7: Comparison of $\frac{\Delta_{soil}}{\Delta_{total}}$ results of finite elements versus Equation 3.2.

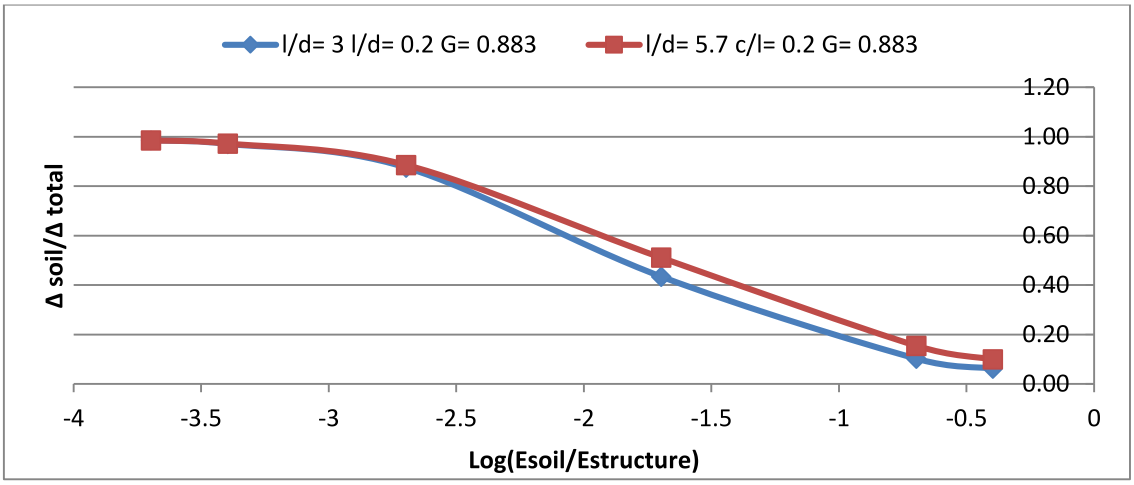

Figure 4.8: The change in the curves with the change of $\frac{l}{d}$ ratio, considering the other variables are constants.

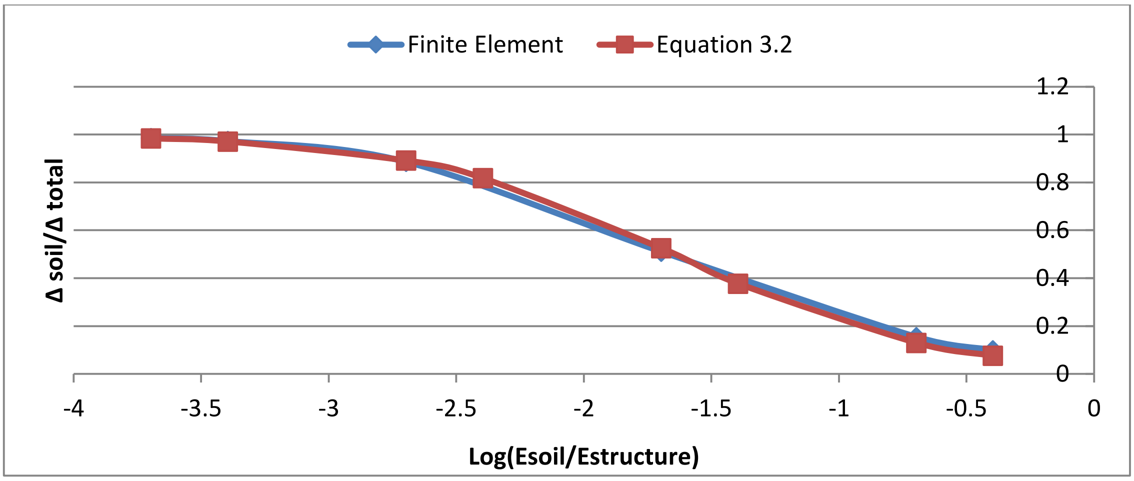

Figure 4.9: Comparison between the finite elements method and Equation 3.2 results for frame with $\frac{c}{l}$ ratio equals 0.2 and $\frac{l}{d}$ value equals 5.7 and G value of 0.833.

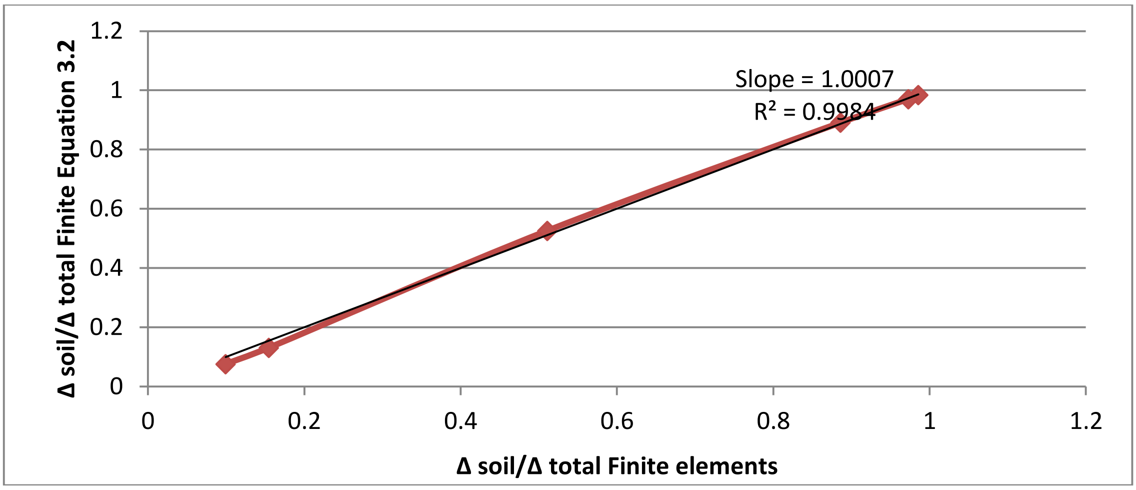

Figure 4.10: Comparison of $\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}}$ results of finite elements versus Equation 3.2.

4.6.2 Effect of beams parameters

To detect the effect of G ratio change, many models with several values of G ratio with the same $\left(\frac{c}{l}\right)$ and $\left(\frac{l}{d}\right)$ ratios are simulated in SAP2000. The results are presented in Figure 4.11, which shows many curves for various values of G ratio, with the same $\left(\frac{c}{l}\right)$ and $\left(\frac{l}{d}\right)$ ratios.

It is obvious from Figure 4.11 that the curves are very close to each other, and that the values are almost identical, which indicates that the effect of changing G ratio on the displacement ratios is insignificant and can be neglected. Although, the importance of G factor on the displacement values will be discussed in the next section.

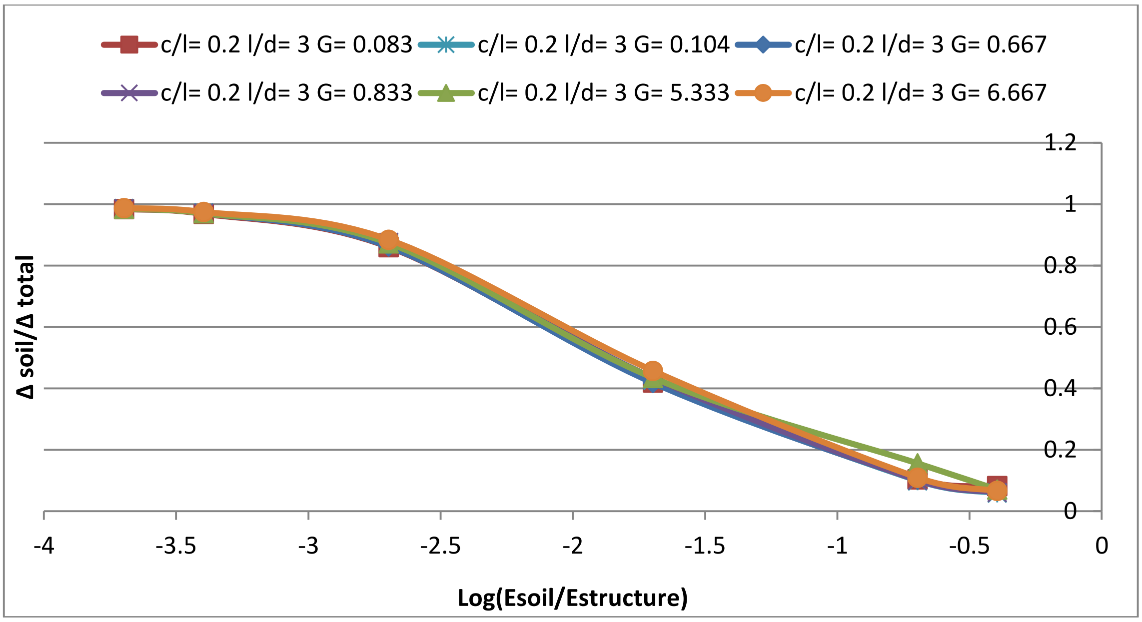

Figure 4.11: The change in the curves with the change of G ratio, considering the other variables are constants.

4.6.3 Factors that affect the displacement of structure

From Table 4.2 and

Table 4.3 it is obvious that the displacements of structure change significantly, where the displacement increases when the soil modulus of elasticity increases. On the other hand, it is obvious from Chapter 3 that the settlement of structure has not changed no matter what the soil modulus of elasticity is.

However, this behavior depends on many factors that affect the displacement of the structure. These factors include the dimensions of the beams, the length of the span and the modulus of elasticity of the soil. The dimensions of the beams and the span length are covered by the G factor, while the modulus of elasticity is governed by two ratios: $\frac{E_1}{E_2}$ which is used to know the limits of the difference between the two soils and $\frac{E_{soil}}{E_{structure}}$, which is used as a reference to reduce the number of variables, and to simplify the observation process, and to have an applicable reference for all models. The obtained displacement of structure for a certain modulus of elasticity of soil $\Delta_{structure}$ will be normalized as ratio by dividing it to the maximum displacement of the structure Δ_{max} obtained from the model with high E_1 value. Thus, the ratio $\frac{\Delta_{structure}}{\Delta_{max}}$ will give an indication of the change in the displacement of structure.

Many models are simulated on the finite elements software, which are used to obtain the displacements of structure. Then, after all the models are tested, and the displacements are recorded, the values of $\frac{\Delta_{structure}}{\Delta_{max}}$ are obtained, and the curve of $\frac{\Delta_{structure}}{\Delta_{max}}$ values for $\frac{E_1}{E_2}$ values is shown in Figure 4.12, taking into consideration that $\left(\frac{l}{d}\right)$ and $\left(\frac{c}{l}\right)$ ratios are assumed constants, and G ratio is variable.

From Figure 4.12 it is obvious that when the G value increases, $\frac{\Delta_{structure}}{\Delta_{max}}$ value increases. As was mentioned earlier, the dimensions of the beams and the span length have great effect on the displacements of the model, where increasing the dimensions of the beams and reducing the span length will increase the stiffness of the beam, thus increasing the moment resistance, which will reduce the middle column's pressure on the soil, especially the weak soil, therefore decreasing the displacement of structure and settlement of soil. Increasing the G ratio means decreasing the dimensions of the beams or increasing the span length, this will reduce stiffness of the beam, thus reducing the effect on the displacement.

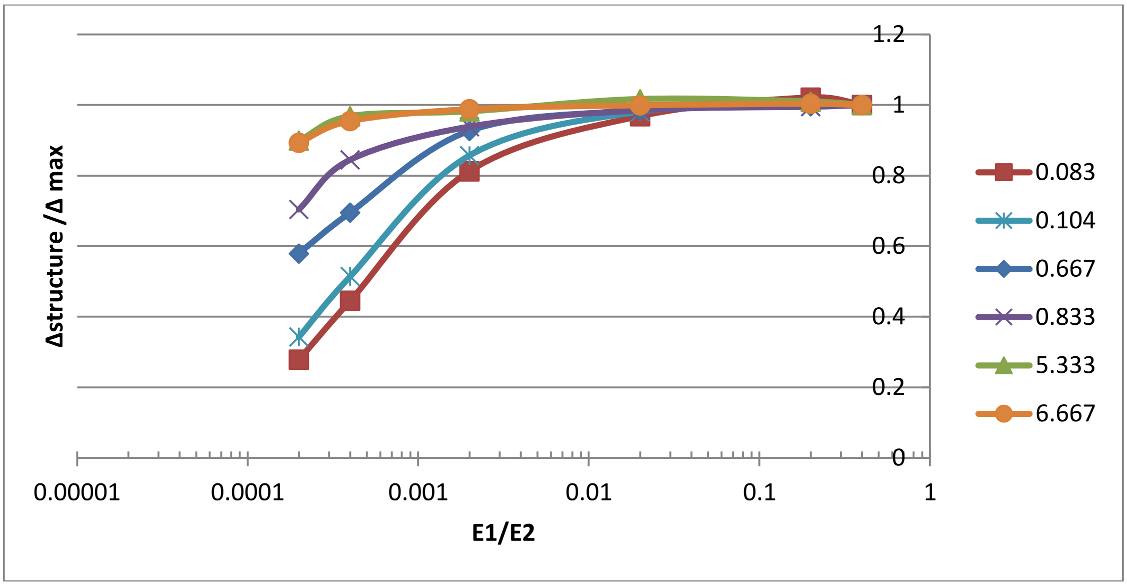

Figure 4.12: The change in $\frac{\Delta_{structure}}{\Delta_{max}}$ for $\frac{E_1}{E_2}$ ratios for several G values.

On the other hand, considering the change of $\frac{E_1}{E_2}$ ratio, it is obvious that when $\frac{E_1}{E_2}$ value increases, $\frac{\Delta_{structure}}{\Delta_{max}}$ increases and approaches 1. This behavior is expected, because increasing the modulus of elasticity of soil will decrease the soil settlement. Likewise, increasing the resistance for the pressure of the structure, which will decrease the beam's share of stress resistance, will cause an increase in the displacement of structure.

It is noticed that for $\frac{E_1}{E_2}$ value of 0.01 or higher, $\frac{\Delta_{structure}}{\Delta_{max}}$ approaches 1. While for $\frac{E_1}{E_2}$ value of less than 0.002, the values of $\frac{\Delta_{structure}}{\Delta_{max}}$ are lower than 1. Although, it must be noted that the ratio $\frac{E_1}{E_2}$ value of less than 0.002 conditions occurred under sever circumstances, it is very rare to have this big difference in the nearby soils, and in such situations soil enhancement must be considered before building. As for the $\frac{E_1}{E_2}$ range between 0.01 and 0.002, the $\frac{\Delta_{structure}}{\Delta_{max}}$ values are approximately between 0.7 and 1, which means that certain reduction factors must be used for the displacement of structure.

Meanwhile, it is safe to use Equation 3.2 for a frame with the same assumptions for $\frac{E_1}{E_2}$ value of 0.01 or higher, and no reduction factors for the soil settlements or displacements of structure are needed.

5 Conclusions and recommendations for further researches

Many points are concluded from the results of this research. Some deals with the general ideas of the research and the fitted equations, the applications and the limitations and others are specific for the frame condition.

5.1 Limitations of the main assumptions

In this research, the main assumptions used in structural and geotechnical engineering were discussed. From the data resulted from the models, the limitation for each assumption is found from the main displacement ratios curves. Therefore, for soil modulus of elasticity ratio of $4 * 10^{-4}$ or less, the flexible soil-rigid structure can be used safely. While, for modulus of elasticity ratio of 0.4 or higher, the rigid soil-flexible structure can be used safely. However, for the modulus of elasticity ratios of $4 * 10^{-4}$ to 0.4, both the soil and structural displacements must be calculated.

As a practical application of the limitations of the assumptions, for the soil types: igneous rocks, limestone, sandstone, shale, dolomite and all the metamorphic rocks, the rigid soil-flexible structure can be safely used. On the other hand, the soil types: very soft and soft clay, silt clay, silt sand and other silt soils, the flexible soil-rigid structure can be used safely.

5.2 The effect of the footing and column dimension ratios

The column and footing parameters were normalized into two dimension ratios, length of footing side to depth of footing $\left(\frac{l}{d}\right)$, length of column side to length of footing side $\left(\frac{c}{l}\right)$. It was noticed in Chapter 3 that these ratios affect the displacement ratios, where increasing $\left(\frac{l}{d}\right)$ and $\left(\frac{c}{l}\right)$ ratios increases the soil displacement ratio and decreases the displacement of structure ratio.

Also, it was concluded that the effect of $\left(\frac{c}{l}\right)$ ratio is more significant than the $\left(\frac{l}{d}\right)$ ratio, where any small change of $\left(\frac{c}{l}\right)$ ratio changes the displacement ratios significantly, while changing $\left(\frac{l}{d}\right)$ ratio gives more close and successive curves.

5.3 The fitted equation

In Chapter 3, the concluded data was used to fit simple equations, in order to obtain the displacement ratios easily. The fitted equations have the logistic function characteristics, with simple sub-equations that govern the variables in the equations.

From Chapter 3 and Chapter 4, the applicability of the fitted equation was discussed, and it was found that the equations give acceptable, reliable displacement ratios that can be used to predict the soil settlement with acceptable accuracy.

5.4 Application of the equation

Because the equations deal with the ratios of the displacement of structure and soil, it can be used in many cases to solve problems and conceptually predict the behavior of the structure and the soil.

First, the equation can be used to predict the displacement of the soil by knowing the displacement of the structure, assuming the foundation is rigid. By using any structural analysis program, the displacement of structure can be obtained, and by knowing the modulus of elasticity of the soil and the structure to total displacement ratio, the total displacement can be found. From knowing the soil to total displacement ratio and total displacement, the soil settlement can be obtained.

Moreover, this method can be used as a footing design method. The designer can choose a suitable displacement ratio and assumes suitable $\left(\frac{l}{d}\right)$. Then, the needed $\left(\frac{c}{l}\right)$ ratio can be found from the curves or the equation, and by knowing the width of the column, the width and the depth of the footing can be found. However, it must be noted that the dimensions of the footing must be checked for the shear and punching shear forces, and the bearing capacity of the soil must be taken into consideration.

5.5 Limitations of the equation

It must be noted that the previously mentioned equations have limitations that must be considered when using them. After many models have been simulated and tested, it was found that the equations cannot predict the behavior and the displacement ratios for the structures with $\left(\frac{c}{l}\right)$ ratios of less than 0.15. Moreover, when $\left(\frac{l}{d}\right)$ values are more than 8, the equations fail to predict the displacements of soil and structure.

5.6 Simple two-span frame condition

Because the materials are assumed to be elastic, isotropic and homogenous, the displacement ratios $\frac{\Delta_{\text{soil}}}{\Delta_{\text{total}}}$ and $\frac{\Delta_{\text{structure}}}{\Delta_{\text{total}}}$ of the frames are almost identical to those from the simple structure.

Despite the displacement ratios are the same as the simple model and the results calculated from the fitted equations, the actual displacements of structure and soil settlement have significant changes when comparing the results with the simple models. These differences are expected due to the stiffness of the beam, which has a share with the footing in resisting the stresses, especially when the soil is weak.

The soil settlements are affected significantly by the beam stiffness. When the modulus of elasticity of the soil is small, the beam resists the external loads, reducing the pressure on the weak soil, and support the column. This behavior reduces the displacement of structure as well the settlements of the soil. The decrease of the displacements depends proportionally on the stiffness of the beam. However, increasing the modulus of elasticity of soil will increase the displacement of structure, where increasing the modulus of elasticity increases the soil resistivity to the external pressures, thus decreasing the beam share of resisting the stresses, because the axial stiffness for the axial members are significantly larger than the moment stiffness of the beam.

As was noticed in Chapter 4, the structure displacements were normalized into the ratio of $\frac{\Delta_{structure}}{\Delta_{max}}$, where this ratio decreased with the decrease of the beam to column stiffness ratio G. However, it is rare to have two nearby soils with significant differences in modulus of elasticity, and as was stated before in Chapter 4, the difference of the displacements is reduced when the modulus of elasticity of the middle soil increased, and for the ratio $\frac{E_1}{E_2}$ value of 0.01 or higher the differences became insignificant, and can be ignored safely.

5.7 Further researches

As stated before, the assumptions used for the materials are elastic isotropic and homogenous. However, neither the concrete nor the soil properties are as assumed. Thus, it is recommended to take the plastic nature of the materials into consideration, to produce more accurate results.

Furthermore, the frame models are simple, with no consideration for the number of stories or the beams in neither the third dimension nor the tie beams or the rectangular columns and footings. Therefore, it is recommended to consider these factors in any further researches.

It was concluded that the displacement ratios have no significant changes between the simple model and the frame. Although, if the plastic behavior of the materials is included, it is expected to have certain differences between the frame models and the simple ones.

The frame model was assumed with no tie beams, however, it is predicted to have changes in the displacement ratios because of the tie beams effect. Therefore, the effect of the tie beams on the frame, the displacements and displacement ratios must be studied.

The effect of multi stories frames had not been covered in this research. It is recommended to study the changes occurred duo to adding stories on the proposed frame.

The $\left(\frac{c}{l}\right)$ ratio is more significant than the $\left(\frac{l}{d}\right)$ ratio in the models affected by vertical loads, it is expected to have another behavior for the lateral loads, where $\left(\frac{l}{d}\right)$ will has a major role in the equation.

Finally, the lateral loads are unfortunately not included in this research. Although, it is highly recommended to study the effect of the lateral loads on the displacement ratios, taking into account that to simulate an accurate model for the lateral loads, certain amount of time, computer power and certain expertise in advanced finite elements tools are needed.

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جامعة النجاح الوطنية
كلية الدراسات العليا

معادلة مبسطة لحل تفاعل التربة مع الأساس للقوى الرأسية للإستخدامات العملية

إعداد
فوزي أبو العدس

إشراف
د. عبد الرزاق طوقان
د. محمود دويكات

قدمت هذه الأطروحة إستكمالاً لمتطلبات الحصول على درجة الماجستير في هندسة الإنشاءات بكلية الدراسات العليا في جامعة النجاح الوطنية، نابلس، فلسطين

2016

معادلة مبسطة لحل تفاعل التربة مع الأساس للقوى الرأسية للإستخدامات العملية

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الملخص

يمثل الهبوط غير المتساوي في التربة تحدي كبير للمهندسين الإنشائيين بسبب المشاكل التي تسببها هذه الظاهرة. العديد من التشققات تحدث في الجدران، الأعمدة والأسقف بسبب هذا الهبوط غير المتساوي في التربة. هذه التشققات تتدرج من ناحية الخطورة والتي قد تكون تشققات بسيطة لا تؤثر في قوة المبنى، وقد تصل إلى تشققات خطرة قد تهدد سلامة المبنى والساكنين.

على مدى السنين، قام المهندسون الجيولوجيون بابتكار العديد من الطرق لقياس الهبوط في التربة. وعلى الرغم من ذلك، فإن هذه الطرق تحتاج إلى خبرات ومعرفة معينة في خصائص وحالات التربة، والتي يفتقر اليها المهندس الإنشائي. لذلك، وبسبب أهمية تفاعل التربة مع الأساس، جاء هذا البحث ليركز على إمكانية الوصول إلى معادلة مبسطة لتقدير الهبوط في التربة بدقة مقبولة، وذلك للإستخدامات العملية كالتصميم والتقييمات الميدانية.

لتسهيل عملية تقدير الهبوط في التربة، سيتم إستخدام نسب الهبوط كمرجع رئيسي لتكوين المعادلة، حيث تم الإفتراض بأن الهبوط في التربة والهبوط في المنشأ مترابطان بحساب نسبتهما للهبوط الكلي، والذي يمثل مجموع الهبوط في التربة والمنشأ. بمعرفة الهبوط في المنشأ ونسبة الهبوط في المنشأ للهبوط الكلي، يتم حساب الهبوط في التربة، والعكس صحيح.

خلال هذا البحث، سيتم إختبار تطبيق الفرضية المذكورة سابقاً على منشأ بسيط يتكون من عمود وقاعدة مربعين، بالإضافة إلى هيكل يتكون من بحرين، وبأبعاد متساوية للأعمدة والقواعد والجسور.

تم إستخدام طريقة العناصر المحدودة في حساب الهبوط في التربة والمنشأ، وللتأكد من الحصول على دقة مقبولة في النتائج، تم تمثيل المنشأ والتربة كعناصر ثلاثية الأبعاد، وبأبعاد وتقسيم معين للحصول على نتائج دقيقة.

سيتم تكوين المعادلة من النتائج المستخرجة من طريقة العناصر المحدودة، وسيتم مناقشة النتائج من خلال عمل مقارنات بين النتائج التي تم الحصول عليها من طريقة العناصر المحدودة والنتائج المحسوبة من المعادلة.